Rensselaer Polytechnic Institute
Troy, New York

A Digital Coherent Memory Filter for Saturn V Bending Mode Identification

Final Report Part D

Grant No. NGR-33-018-~~04~~ 014

National Aeronautics and
Space Administration

by

Brian Hendrickson

Submitted on behalf of

Rob Roy
Associate Professor of Electrical Engineering

August 1967

CONTENTS

LIST OF FIGURES

ABSTRACT

It is the intent of this paper to discuss the merits of a digital coherent memory filter as a low frequency spectrum analyzer; the specific application studied is bending mode isolation in a Saturn V/S-IC booster. From the general analog coherent memory filter, a digital system is derived, fully capable of isolating frequencies as low as .8Hz to within 5%.

As a means of substantiating the method of approach, the design is simulated with the aid of a digital computer. To illustrate the behavior of the coherent memory filter in noise, uncorrelated, additive, gaussian noise is introduced by a random number generating subroutine. From these results, specifications for hardware implementation may be predicted.

PART I

INTRODUCTION

Until now, the coherent memory filter, in both analog and digital forms, has been under study as a real time spectrum analyzer for high frequency radar applications. Using the coherent memory filter, radar return data may be processed and displayed in doppler frequency (target velocity) and target range over a wide band of frequencies, with very little equipment[14]. Use of a series of bandpass filters in a complex range-gating scheme, thus becomes obsolete.

This basic concept may also be utilized in low frequency spectrum analysis with quite useful results. Applied to bending mode analysis of a vehicle in flight, wherein the bending modes are isolated by a bank of tuned filters, the coherent memory filter offers great simplications in hardware. By the use of two delays, a reference frequency generator, and two adders, the entire system of notch filters may be simplified. From this rather simple circuitry, mode data may be isolated in amplitude and frequency to within a very small limit of error. In addition, as a consequence of the coherent memory filter characteristics, data from several locations in the vehicle may be processed simultaneously with the same system, on a time multiplexed basis.

As an aid in comprehending the operation of the coherent memory filter, its behavior in both analog and digital forms will be described. These properties explained, a system design will be developed and studied with the aid of computer simulation.

Part II

THEORY

Analog Coherent Memory Filter

Single Sideband Technique

One of the earliest coherent memory filter designs, has the form of that illustrated in Figure 1.[14] It consists of a sampled input $x(nT)$ delayed and summed with previous samples, after having been shifted in frequency by a constant frequency generator and single sideband (SSB) mixer. The generator frequency is an integral multiple of the reciprocal of the delay line length, T, i.e. shifting frequency

$$f_s = \frac{K}{T} \quad K = 1,2,3.....$$

Thus for any particular time t, between nT and $(n+1)T$ the frequency generator shifts the delayed samples in phase by $\phi(t) = 2\pi f_s t$ radians. Because of the selection of the reference frequency f_s, the phase shift is periodic in T seconds (the delay line length). Mathematically, the phase shift $\phi(t)$ may be characterized as follows:

$$\begin{aligned}\phi(t) &= \frac{2\pi K t_s}{T} + \frac{2\pi KnT}{T} \\ &= \frac{2\pi K t_s}{T} + 2\pi Kn\end{aligned} \qquad (1)$$

where $t = t_s + nT$, $0 \leq t_s \leq T$. The quantity t_s represents the time relative to the start of each delay line cycle; its contribution is a constant phase shift $\phi_s = \frac{2\pi K t_s}{T}$. The $2\pi Kn$ term may be dropped, since it represents the periodic nature of $\phi(t)$ and offers no contribution.

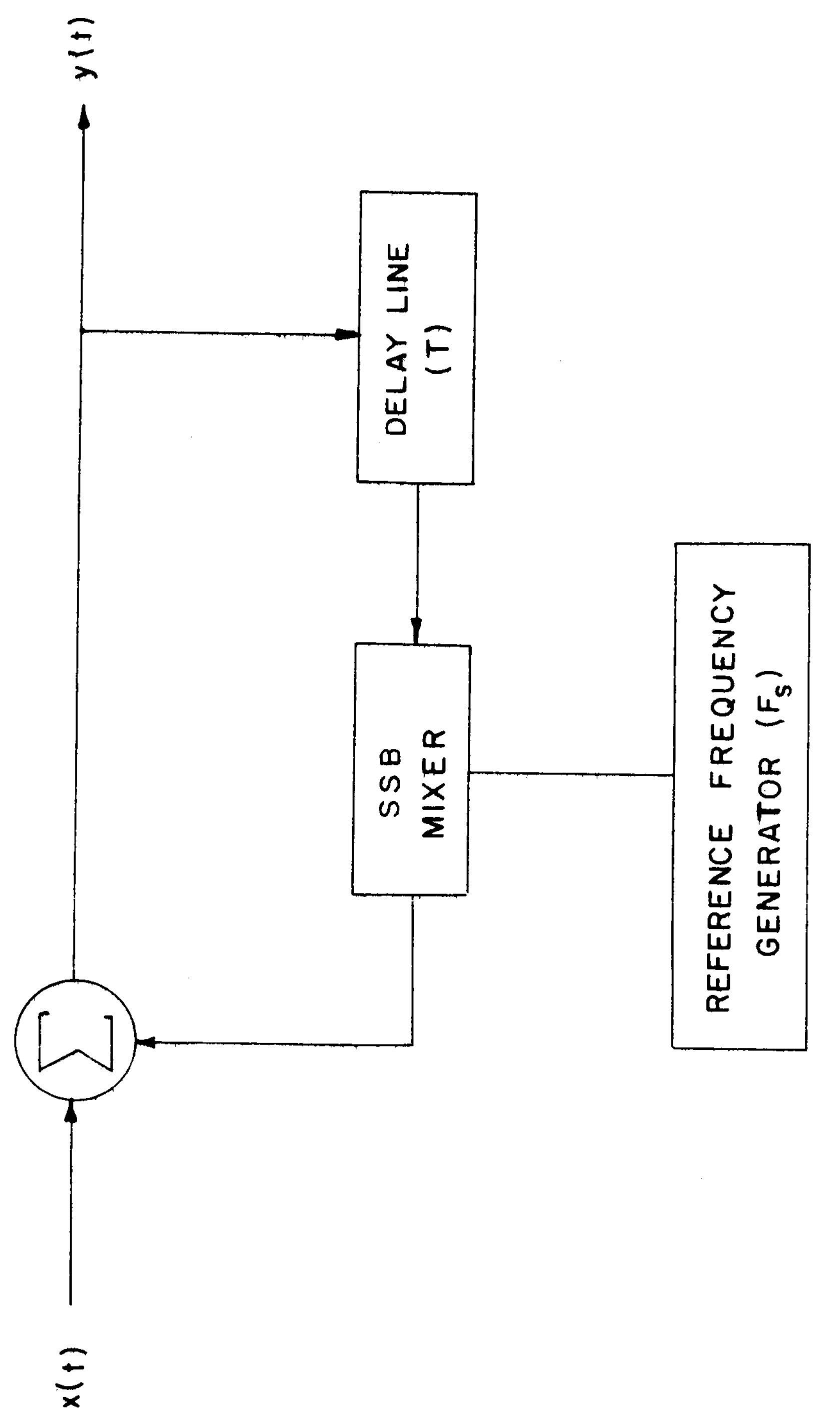

Figure 1 CMF Utilizing SSB Mixer

Therefore:

$$\phi(t) = \frac{2\pi K t_s}{T} = \phi_s \tag{2}$$

From these observations, it follows that the output y_n at time t, after N recirculations, consists of the sample x(t), x(t-T) phase shifted by ϕ_s, x(t-2T) phase shifted by $2\phi_s$ radians, ... and x(t-NT) phase shifted by $N\phi_s$ radians. Mathematically:

$$y_n = \sum_{k=0}^{N} x\left[(n-k)T\right] e^{jk\phi_s} \tag{3}$$

The complex notation denotes the single sideband mixing operation.

If t_s is varied from 0 to T (alternately, ϕ_s varies from 0 to $2\pi k$), the delay line is effectively scanned from 0 to T; the output is seen to resonate at certain points in the delay line. These resonant positions occur where $k\phi_s = wT$ (k=1....N); w is the input radian frequency. The output, which is actually an approximation to the Fourier spectrum of the input, is given in figure 2. Only the positive frequency component is shown, resulting from the complex notation used. Mathematically, it is the following:

$$y(t) = \frac{A}{2} \frac{\sin \frac{N+1}{2}\left[\phi(t_s) - w_oT\right]}{\sin \frac{1}{2}\left[\phi(t_s) - w_oT\right]} e^{j\left[w_ot + \frac{N}{2} \phi(t_s) - w_oT\right]} \tag{4}$$

from an input time function of the form:

$$x(t) = A \cos w_o t$$

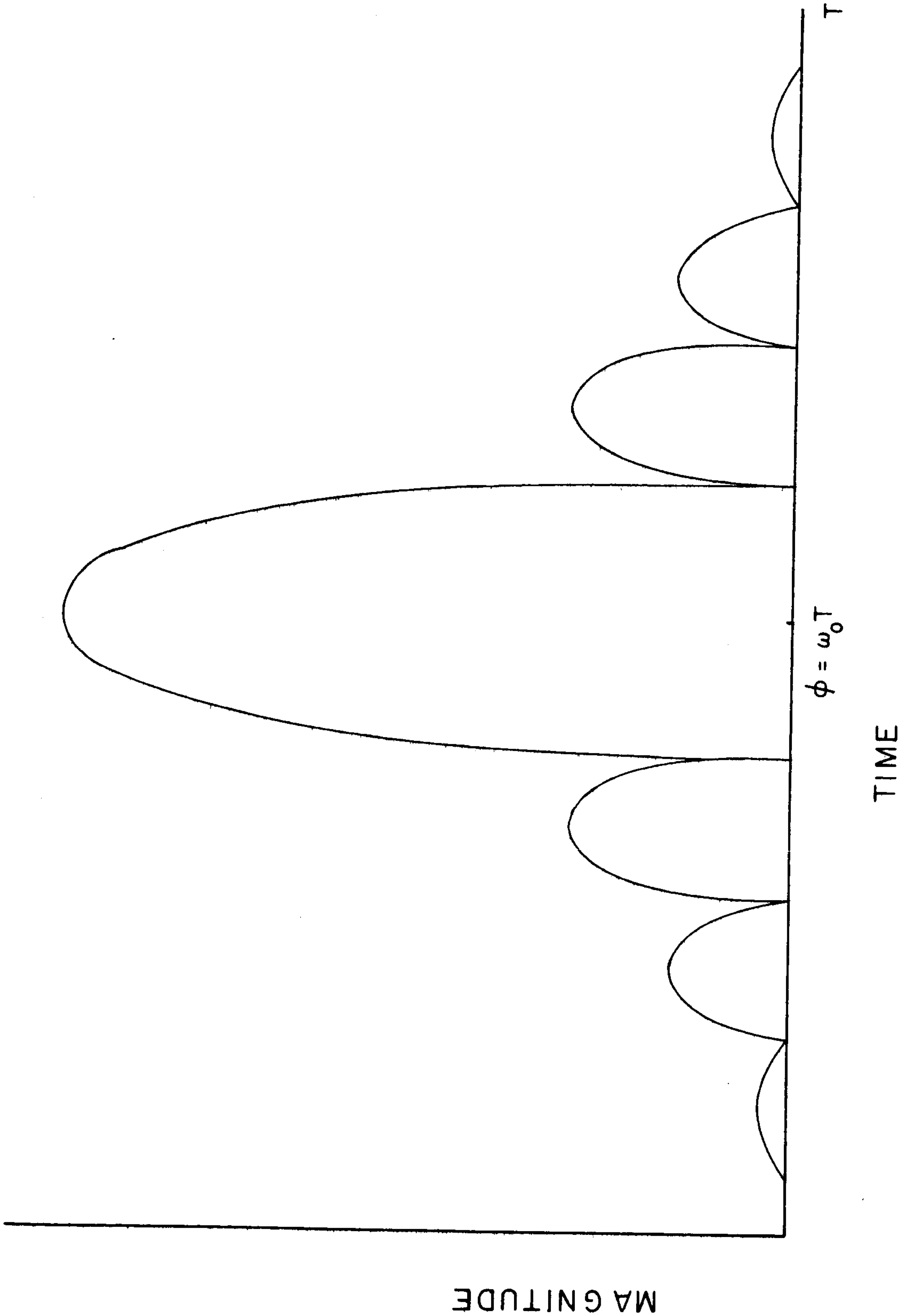

Figure 2 Typical Analog Output

$\mathscr{F}\left[x(t)\right] = \frac{A}{2}\,\delta(w-w_o)$ the positive Fourier transform of $x(t)$.

A coherent memory filter utilizing this theory would require a very stable reference generator and a highly accurate single sideband mixer. These requirements could be achieved at high frequencies; however for very low frequency applications, the technique is not useful.[8,11]

Correlation Scheme

An alternate approach, which is linear in the loop, is shown in figure 3. With this method it is possible to approximate the Fourier spectrum of the input. Consider the equation for the Fourier transform of an arbitrary function x(t):

$$F(f) = \mathscr{F}[x(t)] = \int_{-\infty}^{\infty} x(t)\ e^{-j2\pi ft}\,dt$$

$$= \int_{-\infty}^{\infty} x(t)\ \cos 2\pi ft\ dt - j\int_{-\infty}^{\infty} x(t)\ \sin 2\pi ft\,dt \tag{5}$$

A useful simplication of this equation would be to sample x(t), thereby reducing integration to summation. Responses to cos $2\pi ft$ and sin $2\pi ft$ could then yield an approximation to the spectrum magnitude:

$$\left|F(f)\right|^2 = \sum_{k=0}^{N}\left[x(kT)\ \cos\ 2\pi ft\right]^2 + \sum_{k=0}^{N}\left[x(kT)\ \sin\ 2\pi ft\right]^2 \tag{6}$$

By varying f as shown in figure 4, the above mathematics may be realized physically. Two delay lines thus become necessary, for separate sine and cosine processing. In this system the input samples are correlated with a series of reference frequencies; the reference frequencies

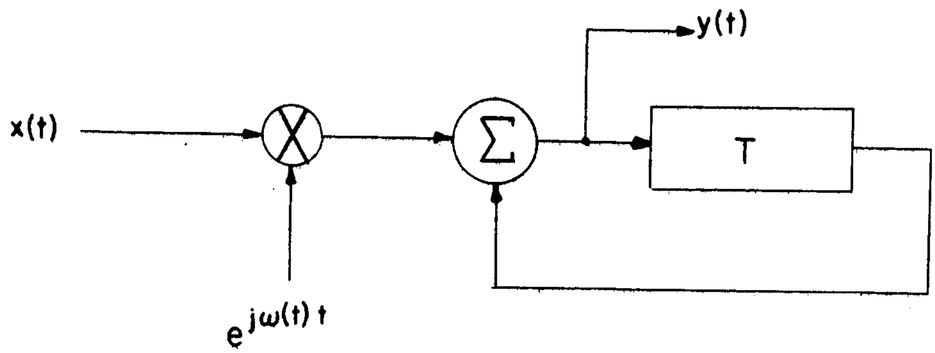

Figure 3 Simplified Correlation Scheme

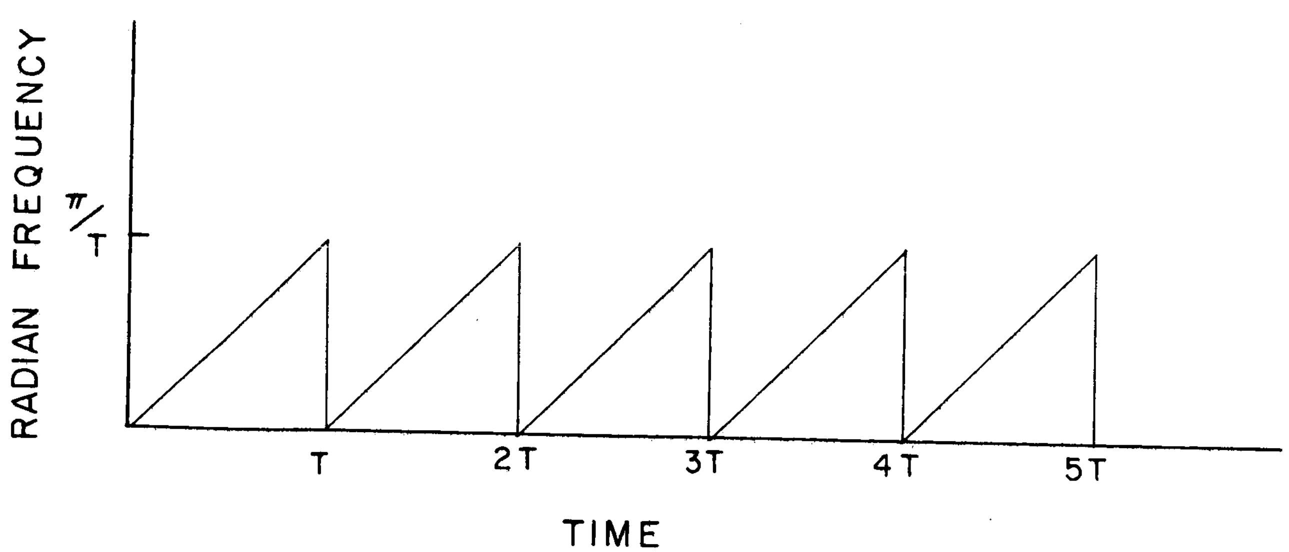

Figure 4 Frequency Sweep

vary continuously with period T. After N recirculations in the delay line the output has the same form as that of the single sideband technique shown in figure 2; however in the correlation scheme the peak corresponds to the reference frequency yielding the largest correlation with the input frequency. Since the reference frequency generation is continuous, the peak occurs exactly at the input frequency. These results may be further explained as follows:

Consider the output using the correlation scheme if cosine correlation only is used:

$$y(t) = \sum_{k=0}^{N} x(t-kT) \cos\left[w(t-kT)\ (t-kT) \right] \qquad (7)$$

where $\quad w(t) = \frac{\pi t}{T^2} \quad$ periodic in T

and $x(t) = A \cos (w_o t + \phi_o)$

Equation (7) upon simplification (see Appendix A), has the following form:

$$y(t) = \frac{A}{2} \frac{\sin \frac{N+1}{2} (-w_o T + \frac{\pi t}{T})}{\sin \frac{1}{2} (-w_o T + \frac{\pi t}{T})} \cos -\phi(t) \qquad (8)$$

$$+ \frac{A}{2} \frac{\sin \frac{N+1}{2} (w_o T + \frac{\pi t}{T})}{\sin \frac{1}{2} (w_o T + \frac{\pi t}{T})} \cos \phi(t)$$

where $\phi(t) = w_o t + \frac{\pi t^2}{T^2} - \frac{N}{2} w_o T - \frac{N\pi}{2T} t + \phi_o$

The above equation is periodic in T and in frequencies differing by $\frac{1}{2T}$. Maxima equal to $\frac{A}{2}$ (N+1) cos ϕ_o are located at the following positions in time

$$t = \frac{w_o T^2}{\pi} \tag{9a}$$

and

$$t = -\frac{w_o T^2}{\pi} \tag{9b}$$

The maximum described by equation (9b) is never seen since the reference frequencies are positive only. If sine correlation were also used the resulting magnitude output would be the following:

$$\left|y(t)\right| = \frac{A}{2} \left| \frac{\sin \frac{N+1}{2} (-w_o T + \frac{\pi t}{T}}{\sin \frac{1}{2} (-w_o T + \frac{\pi t}{T}} \right| \tag{10}$$

As N is increased equation (10) approaches a unit impulse centered at $t = \frac{w_o T^2}{\pi}$. This is the true positive frequency spectrum of a cosine wave of phase ϕ_o existing for all time. Thus for large N, approximations to any input spectrum may be displayed (see Appendix B).

Digital Coherent Memory Filter

The digital realization of the coherent memory filter utilizes the basic concepts of the analog case; in fact the correlation technique is simpler to implement digitally. The more complex, variable phase scheme is extremely inaccurate at low frequencies. For this reason the correlation technique shall be the only method pursued in depth.

The digital correlation scheme yields a discrete Fourier spectrum. Accuracy of display is dependent on the number of frequencies used for reference. A discrete Fourier spectrum would have the following form:

$$\left|F(f_n)\right|^2 \cong \sum_{k=0}^{N}\left[x(kT)\cos 2\pi f_n kT\right]^2 + \sum_{k=0}^{N}\left[x(kT)\sin 2\pi f_n kT\right]^2 \tag{11}$$

where f_n is the n^{th} reference frequency; kT, the k^{th} sampling interval.

The above equation describes what a digital coherent memory filter does. Each frequency f_n is supplied once per period in the form of $\cos 2\pi f_n kT$ and $\sin 2\pi f_n kT$. The correlations of each frequency with the input sample are summed with previous correlations of the same frequency and stored as separate spectral components. If M of these frequencies are supplied in period T the total storage in the delay line represents an approximation to a discrete Fourier spectrum from 0 to $\frac{1}{2T}$ Hz. In radar applications, correlations of the same frequency are not summed but are maintained separately in the delay line, yielding both range and doppler information.

Figure 5 illustrates the transform procedure in matrix form. A discrete Fourier transform would perform straight matrix multiplication as illustrated, i.e. correlation of a single frequency with all time samples. The digital coherent memory filter multiplies each sample X_{kT} by the column (kT), storing each word separately in the delay line, i.e. correlation of a single time sample with all frequency components. Either method yields the same result, but the coherent memory filter processes the information faster. (For complete derivation see appendix C).

$$
\begin{array}{cc}
 & \begin{array}{cccccc} 0 & T & 2T & \cdots & & NT \end{array} \\
\begin{array}{c} 0 \\ \omega \\ 2\omega \\ 3\omega \\ \vdots \\ M\omega \end{array} &
\begin{bmatrix}
1 & 1 & 1 & \cdots & & 1 \\
1 & \cos\theta & \cos 2\theta & \cdots & & \cos N\theta \\
1 & \cos 2\theta & \cos 4\theta & \cdots & & \cos 2N\theta \\
1 & \cos 3\theta & \cos 6\theta & \cdots & & \cos 3N\theta \\
\vdots & \vdots & \vdots & & & \vdots \\
1 & \cos M\theta & \cos 2M\theta & \cdots & & \cos MN\theta
\end{bmatrix}
\end{array}
\begin{bmatrix} x_0 \\ x_T \\ x_{2T} \\ x_{3T} \\ \vdots \\ x_{NT} \end{bmatrix}
=
\begin{bmatrix} F_1(0) \\ F_1(\omega) \\ F_1(2\omega) \\ F_1(3\omega) \\ \vdots \\ F_1(M\omega) \end{bmatrix}
$$

$$
\begin{array}{cc}
 & \begin{array}{cccccc} 0 & T & 2T & \cdots & & NT \end{array} \\
\begin{array}{c} 0 \\ \omega \\ 2\omega \\ 3\omega \\ \vdots \\ M\omega \end{array} &
\begin{bmatrix}
0 & 0 & 0 & \cdots & & 0 \\
0 & \sin\theta & \sin 2\theta & \cdots & & \sin N\theta \\
0 & \sin 2\theta & \sin 4\theta & \cdots & & \sin 2N\theta \\
0 & \sin 3\theta & \sin 6\theta & \cdots & & \sin 3N\theta \\
\vdots & \vdots & \vdots & & & \vdots \\
0 & \sin M\theta & \sin 2M\theta & \cdots & & \sin MN\theta
\end{bmatrix}
\end{array}
\begin{bmatrix} x_0 \\ x_T \\ x_{2T} \\ x_{3T} \\ \vdots \\ x_{NT} \end{bmatrix}
=
\begin{bmatrix} F_2(0) \\ F_2(\omega) \\ F_2(2\omega) \\ F_2(3\omega) \\ \vdots \\ F_2(M\omega) \end{bmatrix}
$$

$$\theta = \omega T$$

$$F(n\theta) = \sqrt{F_1(n\theta)^2 + F_2(n\theta)^2}$$

Figure 5 Discrete Fourier Transform Using A Digital CMF

Behavior in Noise

Consider the response of the coherent memory filter to input noise alone; the noise power spectrum is assumed bandlimited to $W(W >> \frac{1}{T})$. Since W is large with respect to $\frac{1}{T}$, the noise samples are uncorrelated. As discussed earlier, the coherent memory filter attempts to correlate all input data with a series of reference frequencies to obtain an approximation to a Fourier spectrum of the input in the band $-\frac{1}{2T}$ to $+\frac{1}{2T}$. Because the noise samples are incorrelated in T, they will offer little contribution to the spectrum. In fact, if an input is corrupted by uncorrelated noise, its signal to noise ratio will be improved by a factor of $N+1$.[13] The major effect of noise may be shown to increase the sidelobes of the output function (see appendix D).

PART III

DESIGN PROCEDURE

To apply the correlation technique to low frequencies, it is necessary to examine the theory in more detail. In this section the system is studied closely to achieve simplicity, emphasizing the hardware requirements. The resulting system is shown in Figure 9.

Magnitude Detection

As discussed earlier, it is desired to approximate the correlation of the input with $e^{j2\pi ft}$ by $\cos 2\pi f_n kT + j\sin 2\pi f_n kT$. The system therefore requires separate correlation of reference sines and cosines; the envelope of the input spectrum will only be displayed, disregarding any constant phase shift. Two delay lines are therefore required.

If the input is a series of sinusoids of arbitrary phase, i.e.

$$x(t) = A_1 \cos(w_1 t + \phi_1) + A_2 \cos(w_2 t + \phi_2) + A_3 \cos(w_3 t + \phi_3)$$

$$+ A_4 \cos(w_4 t + \phi_4) \qquad w_1 < w_2 < w_3 < w_4 \qquad (12)$$

maxima occurring in the delay line, using cosine correlation, will closely approximate $\frac{A_1}{2}(N+1)\cos\phi_1$, $\frac{A_2}{2}(N+1)\cos\phi_2$, $\frac{A_3}{2}(N+1)\cos\phi_3$, and $\frac{A_4}{2}(N+1)\cos\phi_4$, respectively. The delay line for sine correlation will yield maxima corresponding to $\frac{-A_1}{2}(N+1)\sin\phi_1$, $\frac{-A_2}{2}(N+1)\sin\phi_2$, $\frac{-A_3}{2}(N+1)\sin\phi_3$, and $\frac{-A_4}{2}(N+1)\sin\phi_4$. Further processing yields estimates of A_1, A_2, A_3, and A_4 at w_1, w_2, w_3, and w_4.

These results show that, if desired, approximations to the constant input phases may be obtained by comparing the output magnitude with the maxima in the delay line; this yields an approximation to sin ϕ_i and cos ϕ_i, where ϕ_i is the input phase. Thus, each mode may be described unambiguously:

$$\text{Mode}_i(t) = A_i \cos 2\pi f_i t \cos\phi_i - A_i \sin 2\pi f_i t \sin\phi_i \tag{13}$$

$$\cos \phi_i = \frac{2F_i(f_i)}{A_i} \tag{14a}$$

$$\sin \phi_i = - \frac{2\, F_2(f_i)}{A_i} \tag{14b}$$

where $F_i(f_i)$ and $F_2(f_i)$ are the spectral estimates stored after cosine and sine correlation respectively.

Input Sampling Rate

Because each sample of the input is to be correlated with each of M frequencies, it is only necessary to sample every T seconds, the delay line length. The envelope of the output described by equation (10) is periodic in frequencies differing by $\frac{1}{2T}$. This result is a consequence of the foldover at the half sampling frequency present in the frequency spectrum of any sampled data system. Thus only frequencies between $\pm \frac{1}{2T}$ may be processed unambiguously. The maximum frequency to be processed is 8.0 Hz, as prescribed by the data in Table 1 - XI Boeing Report No. D5-15381-1. Therefore the delay line length T must be less than .0625 seconds for unambiguous processing:

$$f_{max} \leq 8.0 \text{ Hz}$$

$$\leq \frac{1}{2T}$$

$$T \leq \frac{1}{2f} = .0625 \text{ seconds}$$

Number of Data Recirculations, N

Considerations on N are dependent upon the time allowed to process the data and the improvement in signal to noise ratio desired. Process time is governed by the variations in mode frequencies during the flight and the sharpness of the output maxima necessary for proper frequency isolation.

The frequency data, given in Boeing Report No. D5-15381-1, shows a maximum frequency variation with time of .2Hz/sec; this occurs in mode 4 during the last few seconds of flight (see appendix E). The mode frequencies remain essentially constant at the beginning of the flight; therefore if the process time is selected for worst case, there will be very small errors incurred at the lower frequencies. For a worst case error of 10% on the fourth mode, process time should be a maximum of 2.8 seconds.

Any time limited signal has an infinite frequency spectrum; the smaller the duration in time, the flatter and broader the spectrum. To maintain the spectral representation unambiguous, such that no two mode frequencies overlap, it becomes necessary to select a large number of recirculations, N. For 2.8 seconds process time, a maximum N of 45 may be selected, which yields unambiguous representation of mode frequencies which differ by no less than .36 Hz. (see appendix F).

Worst case errors due to frequency variation then become:

MODE 1: 1.03%

MODE 2: .47%

MODE 3: .75%

MODE 4: 10.0 %

Figure 6 illustrates the magnitude output with increasing N. As N grows large, the peaks become more sharply defined. For N = 45, it is possible to isolate modes differing by .36 Hz.

Reference Frequency Generator

Because spectrum magnitudes are being displayed, it is necessary to generate both reference cosines and sines from 0 to 8.0 Hz. Each of these reference values will be used every T seconds, corresponding to the sampling interval. As discussed before, this scheme performs a correlation for each reference frequency as shown below:

$$F_1(f_n) = \sum_{k=0}^{N} x(kT)\cos(2\pi f_n kT) \qquad \text{cosine correlation}$$

$$F_2(f_n) = \sum_{k=0}^{N} x(kT)\sin(2\pi f_n kT) \qquad \text{sine correlation}$$

Thus after correlation each reference value must be adjusted to a new value to be used T seconds later as shown in the equations above. This

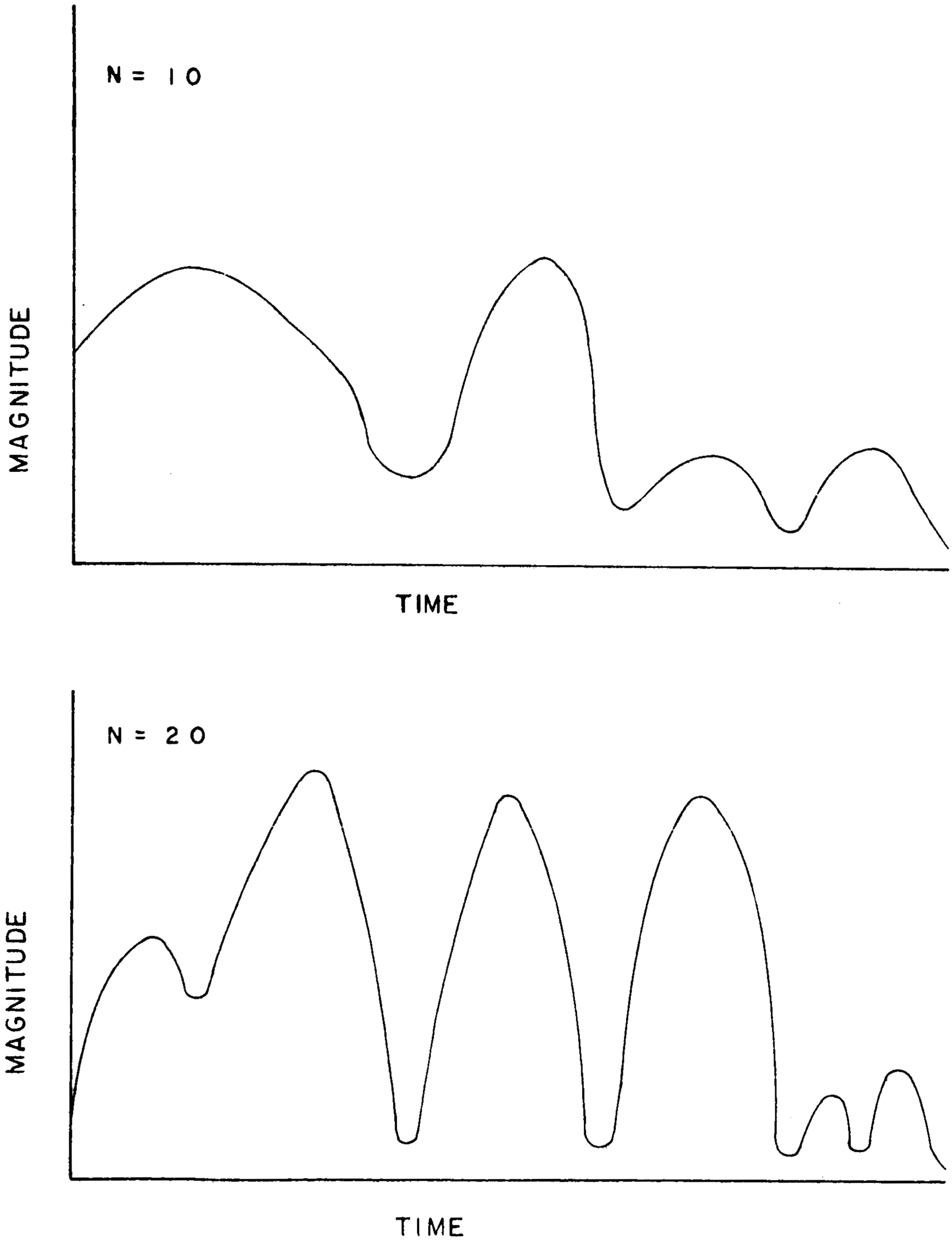

Figure 6 Variation of Output with N

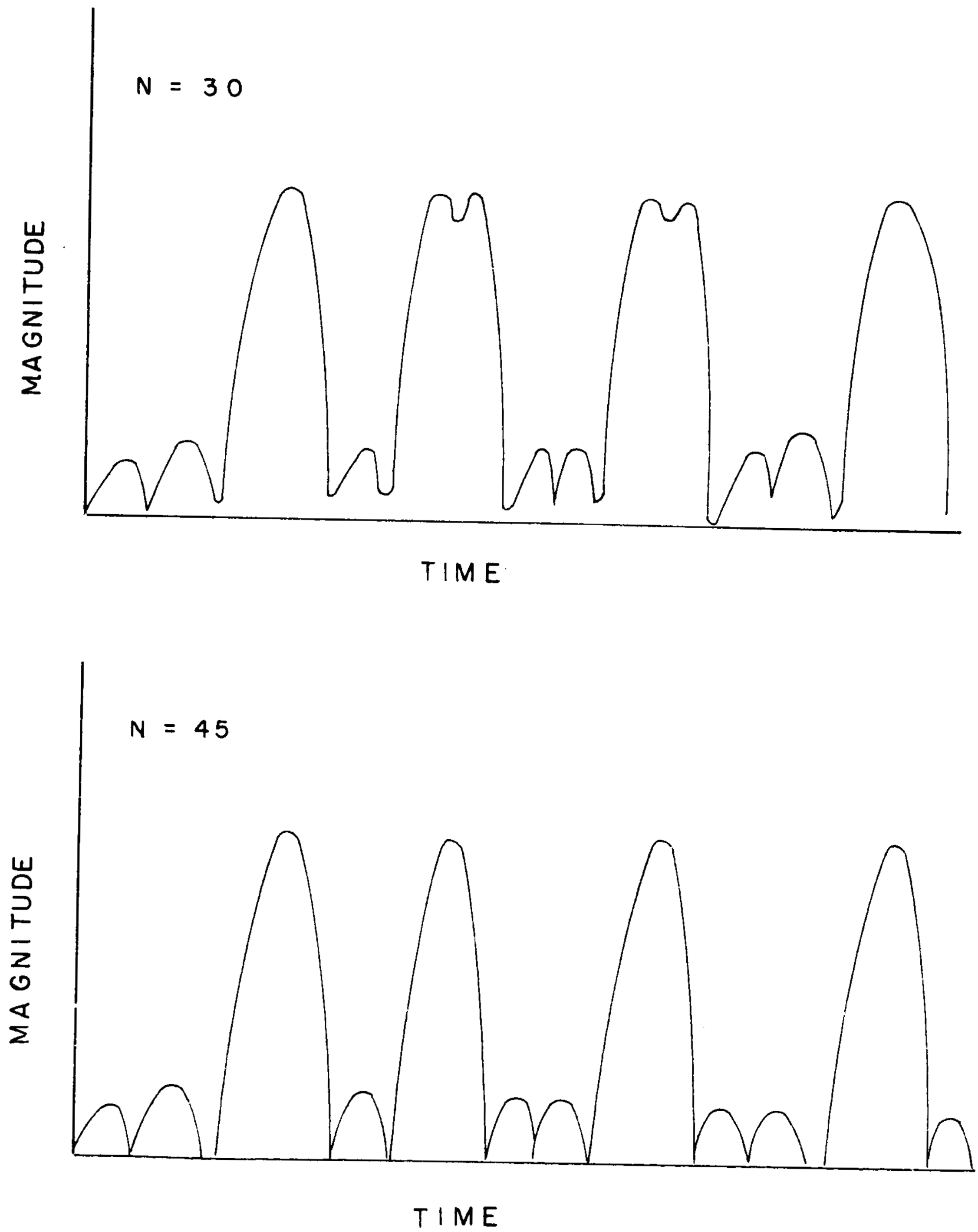

Figure 6 (cont'd)

may be accomplished digitally by the following algorithm:

$$\cos(2\pi f_n kT = \cos\left[2\pi f_n (k-1)\, T\right] \cos 2\pi f_n T \tag{15a}$$

$$-\sin\left[2\pi f_n (k-1)\, T\right] \sin 2\pi f_n T \qquad n=1.....M$$

$$\sin(2\pi f_n kT) = \cos\left[2\pi f_n (k-1)\, T\right] \sin 2\pi f_n T \tag{15b}$$

$$+ \sin\left[2\pi f_n (k-1)\, T\right] \cos 2\pi f_n T \qquad n=1.....M$$

where $\sin 2\pi f_n T$ and $\cos 2\pi f_n T$ are constants. All reference sines are initialized to 0.0, and all reference cosines to 1.0 at the beginning of the process.

If frequencies are to be isolated to within 5%, .8 Hz being the lowest frequency, 201 reference sines and cosines each differing by .04 Hz must be used for correlation. This further requires that $\cos 2\pi f_n T$ and $\sin 2\pi f_n T$ differ by at least one binary digit from $\cos 2\pi f_{n+1} T$ and $\sin 2\pi f_{n+1} T$. To achieve this, a ten bit word length for all reference values is required. As a result the frequency generator must be capable of storing 402 ten bit reference values and 402 ten bit constants; the frequency generator must also be capable of computing all correlations and calculations of new reference values within the sampling period, T.

The above method requires two multiplications and one addition for each reference cosine and sine. Accuracy decreases with each operation; however, by a different interpretation of the reference values it is possible to simplify the frequency generator. It will be shown that only 400 ten bit cosines need be stored to generate all reference values.

This method requires storage of cosine values from 0 to $\frac{1}{T}$ in contrast to the first method which generated reference values for frequencies from 0 to $\frac{1}{2T}$; however only half the reference values need be used at any one time. Therefore, the system may still correlate frequencies between 0 and $\frac{1}{2T}$ HZ.[6]

It is obvious from figure 5 that all cosine references for t=0 (column 1) are 1.0, all sines, 0.0. It is not necessary to store M ones and zeros; one of each is all that is needed. Because of the periodic nature of the cosine, each value in the matrix is equivalent to at least one value in column two. Therefore, with a slightly more complex method of selecting reference values, only the second column need be stored.

Consider the generation of cosine reference values:

$$c(m) = \cos\left(\frac{2\pi mkT}{MT}\right) \qquad k=0.....N \qquad (16a)$$

$$m=0.....M$$

$$= \cos\left(\frac{2\pi mk}{M}\right) \qquad (16b)$$

The second column of the matrix in figure 5 contains all values of equation (16) for k=1, m=0.....M. Equation (16) shows that this implies generation of cosine values from 0 to 2π in M steps, which is equivalent to generating frequencies from 0 to $\frac{1}{T}$.

Assume that correlation for t=T has concluded, and it is desired to generate a new set of reference consines Z_m, m=0.....M, for t=2T. The values used for t=T were the values generated in equation (16b):

$$Z_m(1) = c(m) \qquad m=0.....M \tag{17}$$

$$= \cos \frac{2\pi mk}{M} \qquad k=1$$

$$Zm(2) = \cos \frac{2\pi 2m}{M} \tag{18}$$

If $2m \leq M$ the value of $Z_m(2)$ is given by

$$Z_m(2) = c(2m) \tag{19a}$$

If $2m > M$

$$Z_m(2) = c(2m-M) \tag{19b}$$

because of the periodic nature of the cosine in 2π:

$$\cos(2\pi + x) = \cos(2\pi - x)$$

The above procedure is the same for any value k; it is only required that $Z_m(k)$ be known to calculate $Z_m(k+1)$:

$$Z_m(k) = c(i) = \cos\left(\frac{2\pi i}{M}\right) \tag{20}$$

$$Z_m(k) = \cos\left(\frac{2\pi mk}{M}\right) \tag{21}$$

$$Z_m(k+1) = \cos \frac{2\pi m(k+1)}{M} \tag{22}$$

$$= \cos \frac{2\pi(i+m)}{M} \tag{23}$$

$$= c(i+m) \quad i+n \quad M \tag{24a}$$

$$= c(i+m-M) \quad i+n \quad M \tag{24b}$$

Using the same analysis, it is possible to generate all sine values using the same M stored cosines.

$$y_m(k) = \sin\left(\frac{2\pi i}{M}\right) = \cos\left(\frac{2\pi i}{M} - \pi/2\right) \tag{25}$$

$$= \cos\left(\frac{2\pi i}{M} + 3\pi/2\right) \tag{26}$$

$$\therefore \quad y_m(k) = c\left(i - \frac{M}{4}\right) \qquad i - \frac{M}{4} \geq 0 \tag{27a}$$

$$= c\left(i + \frac{3M}{4}\right) \qquad i - \frac{M}{4} < 0 \tag{27b}$$

·if $M/4$ is an integer

A flow chart of this procedure is given in figure 5.

The only requirements for this procedure is that all stored cosines must differ in at least one binary digit; therefore ten bit storage is required.

Hence, by this table look-up procedure, all correlation values may be generated by storage of only 400 ten bit numbers. The only errors will be truncation errors resulting from quantization to ten bit word lengths. In addition this scheme will yield more symmetric approximations to the input spectrum, thus making sidelobe reduction easier.

Sidelobe Suppression

It is possible to reduce the sidelobe response of the output, to achieve better mode isolation, in a number of ways.[14] One of the easiest methods to implement digitally is by the use of a Hanning window.[2,14] By this method, individual spectral components are summed in the following manner:

$$F(f_n) = -.5\,F(f_{n-j}) + F(f_n) - .5\,F(f_{n+j}) \qquad (28)$$

where

$$j = \frac{1/(N+1)T}{.04} = 9$$

When $F(f_n)$ is a maximum, the components $F(f_{n-j})$ and $F(f_{n+j})$ should equal 0.0. Thus the maximum is uneffected, but the sidelobes are reduced.

Since the spectral magnitudes are stored individually it is quite easy to implement equation (28) digitally. All that is required is a

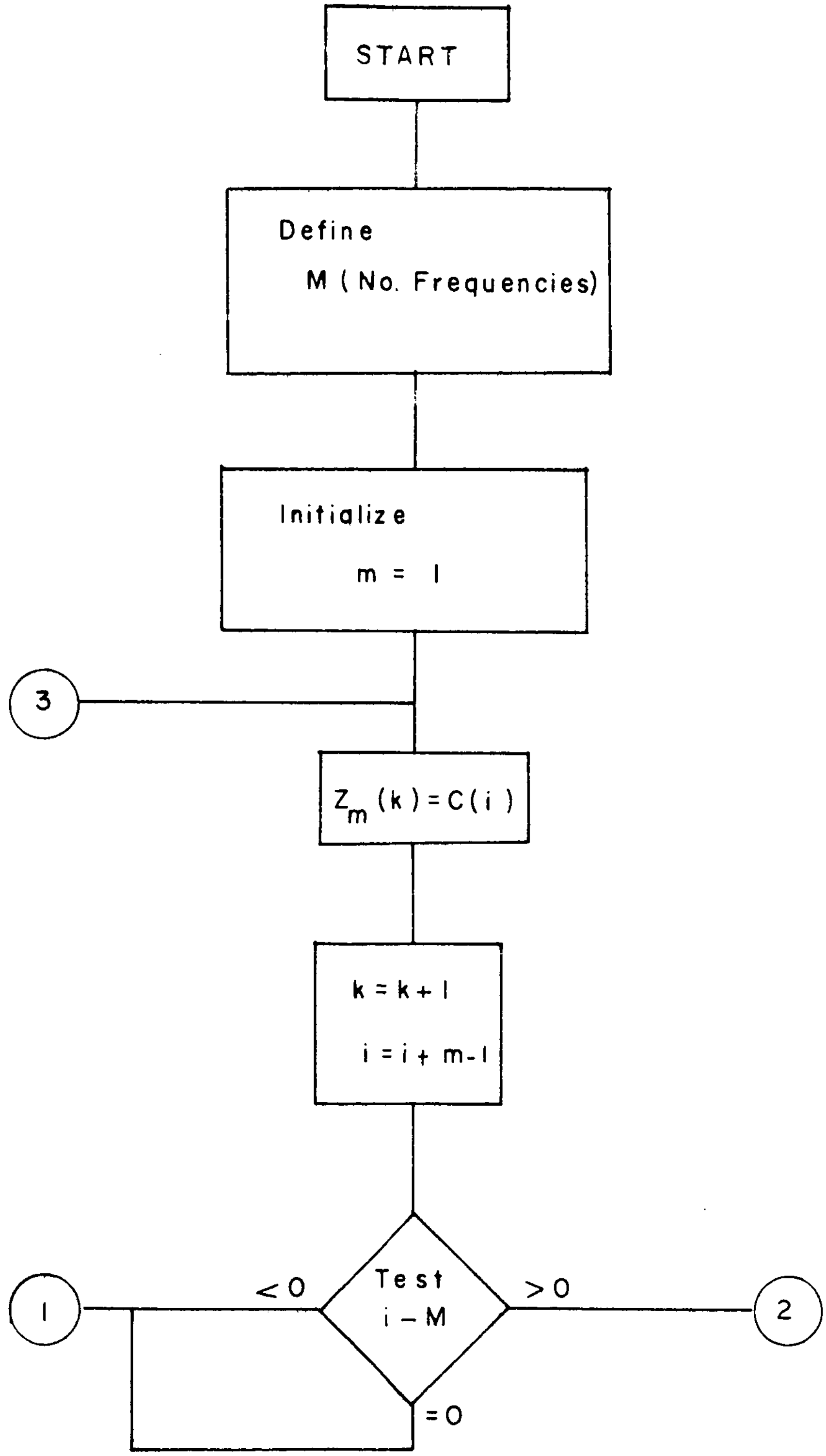

Figure 7 Flow Chart of Table-Lookup Scheme

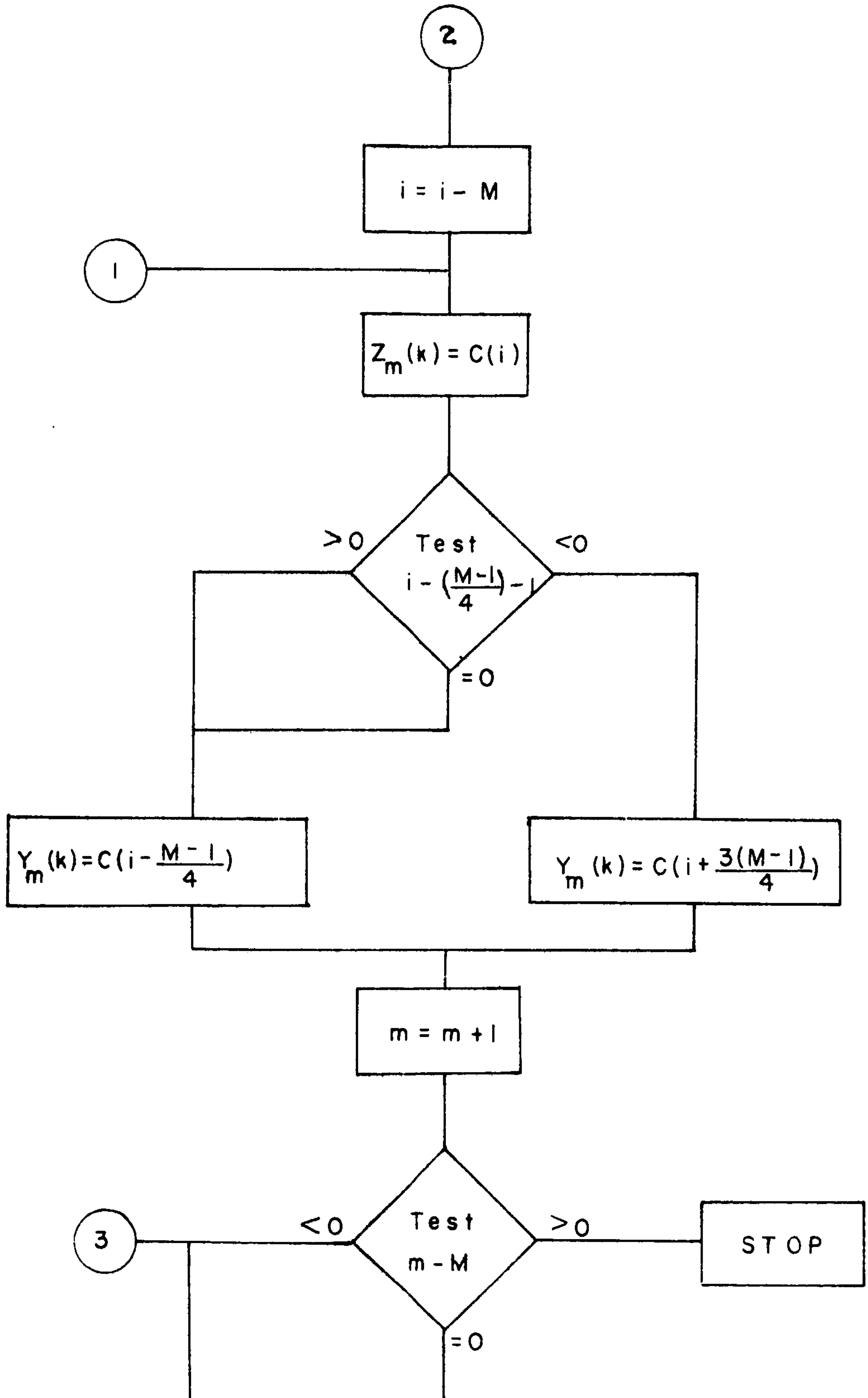

Figure 7 (cont'd)

digital shifting and adding procedure. The first sidelobes are approximately 13db below the maximum; all other sidelobes, approximately 40db below the maximum.

Multiplexing

By the nature of the technique, speed of correlation of the input sample is only limited by the multiply rate. It is therefore possible to correlate samples from different locations of the vehicle in time T on a time multiplexed basis. The same system may be used, the only further requirement being increased storage in the delay line.

Behavior of the Coherent Memory Filter in Noise

It will prove useful to study the behavior of the coherent memory filter in the presence of input noise. A gaussian model will be assumed. All noise samples are assumed uncorrelated with zero mean. The coherent memory filter input then becomes

$$x(kT) = \sum_{i=1}^{4} \text{mode}_i(kT) + n_{kT} \tag{29}$$

where n_{kT} is the noise sample. The probability density function of the noise has the following form:

$$p(\lambda) = \frac{1}{2\pi} \quad \exp\left[-\lambda^2/2\sigma^2\right] \tag{30}$$

where λ has the dimensions of voltage and σ is the standard deviation or rms value of n_{kT}. The signal to noise power ratio becomes:

$$S/N = \frac{\sum_{i=1}^{4} \text{mode}_i(kT)_{rms}}{2\sigma^2} \tag{31}$$

By varying σ and using a gaussian random number generator in the computer simulation, a minimum signal to noise ratio may be specified for proper system operation.

Computer Simulation

The design as developed will be simulated with the aid of a digital computer. Since, by the nature of digital hardware all inputs and outputs are confined to discrete values, it is necessary to quantize and round to discrete values after each operation in the simulation; this is accomplished on the computer by a subroutine. From the computer simulation, it will be possible to generate error curves for frequency and amplitude isolation, and for behavior in noise. A listing of the entire simulation is given in the appendix.

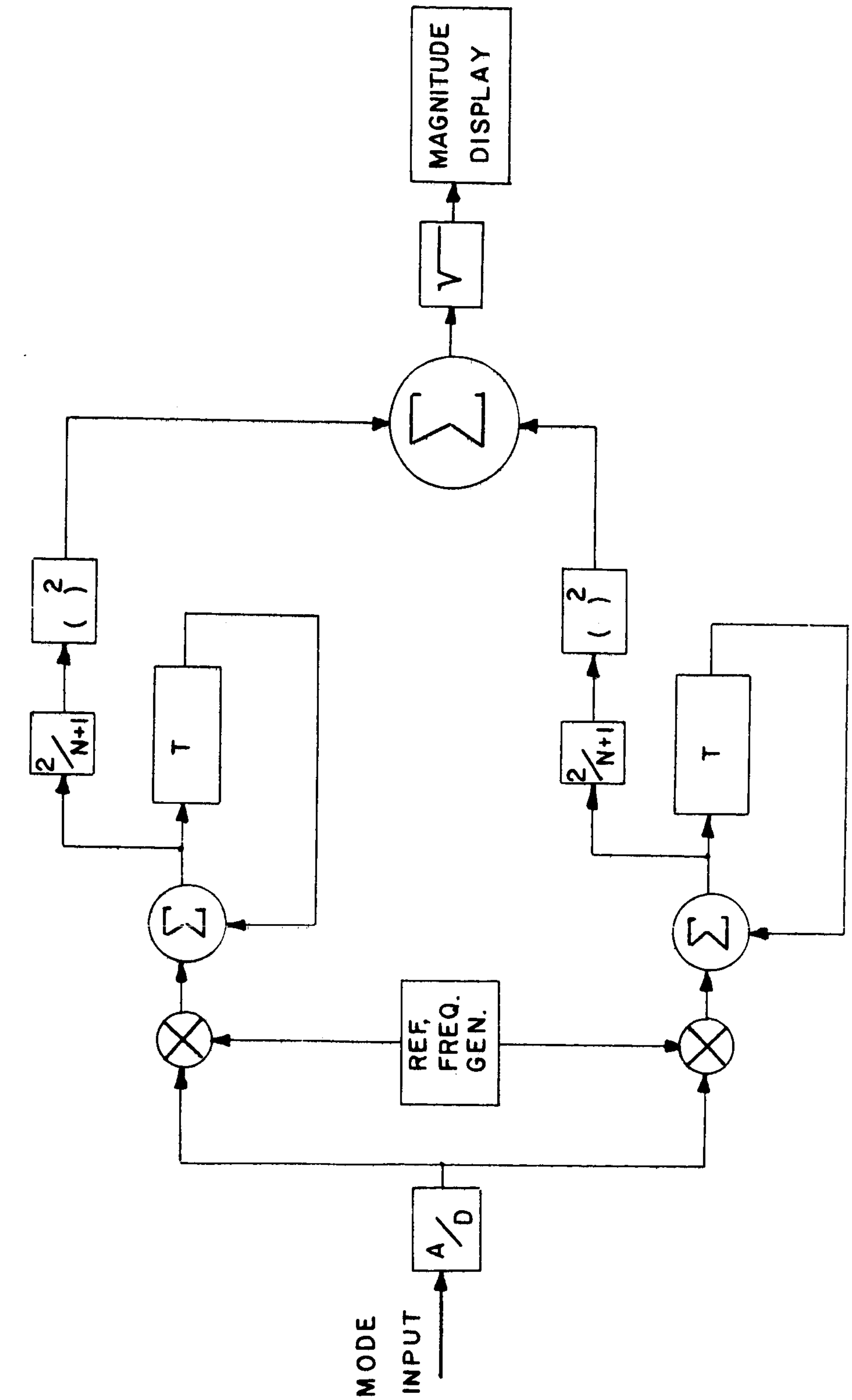

Figure 8 Digital System

PART IV

SIMULATION RESULTS

The system as designed was studied by computer simulation; frequencies studied ranged from .75 Hz to 8.0 Hz; amplitudes, from .05 to 10.0; S/N (power) ratios, from 10 to 30 db. In addition, the phase of each bending mode was varied to test its effect on the ability of the system to display each mode as a separate amplitude and frequency; the imput time function had the form given by equation (12).

The system was first simulated without smoothing the output. Typical errors in amplitude displayed are shown in Figure 9; the minimum detectable mode amplitude was .05. Amplitude errors (%) varied from 10 to 15% in the region .05 to 10.0 as shown. A quantization interval of $1/2^{10}$ and a frequency stepsize of .04 Hz may be expected to yield spectral approximations to within this limit of error.

Frequency errors for mode amplitudes between .05 and 10.0 ranged from 0.0 to 2.0% as shown in Figure 10; this is what should be expected. As discussed in appendix C, resonance occurs at reference frequencies which are closest to the input frequency. For a frequency stepsize of .04 Hz, the maximum frequency error should be .02 Hz or 2.5% at .8 Hz. Any deviation from this limit of error is due to quantization error.

The above results were tested for various input phase shifts from 0 to 2π. The presence of a phase shift, no matter how different from mode to mode, had no appreciable effect on the output; in fact, by using the method discussed in Part III, it was possible to obtain an estimate of each phase shift, in the form of sin ϕ and cos ϕ, to within 20%. The

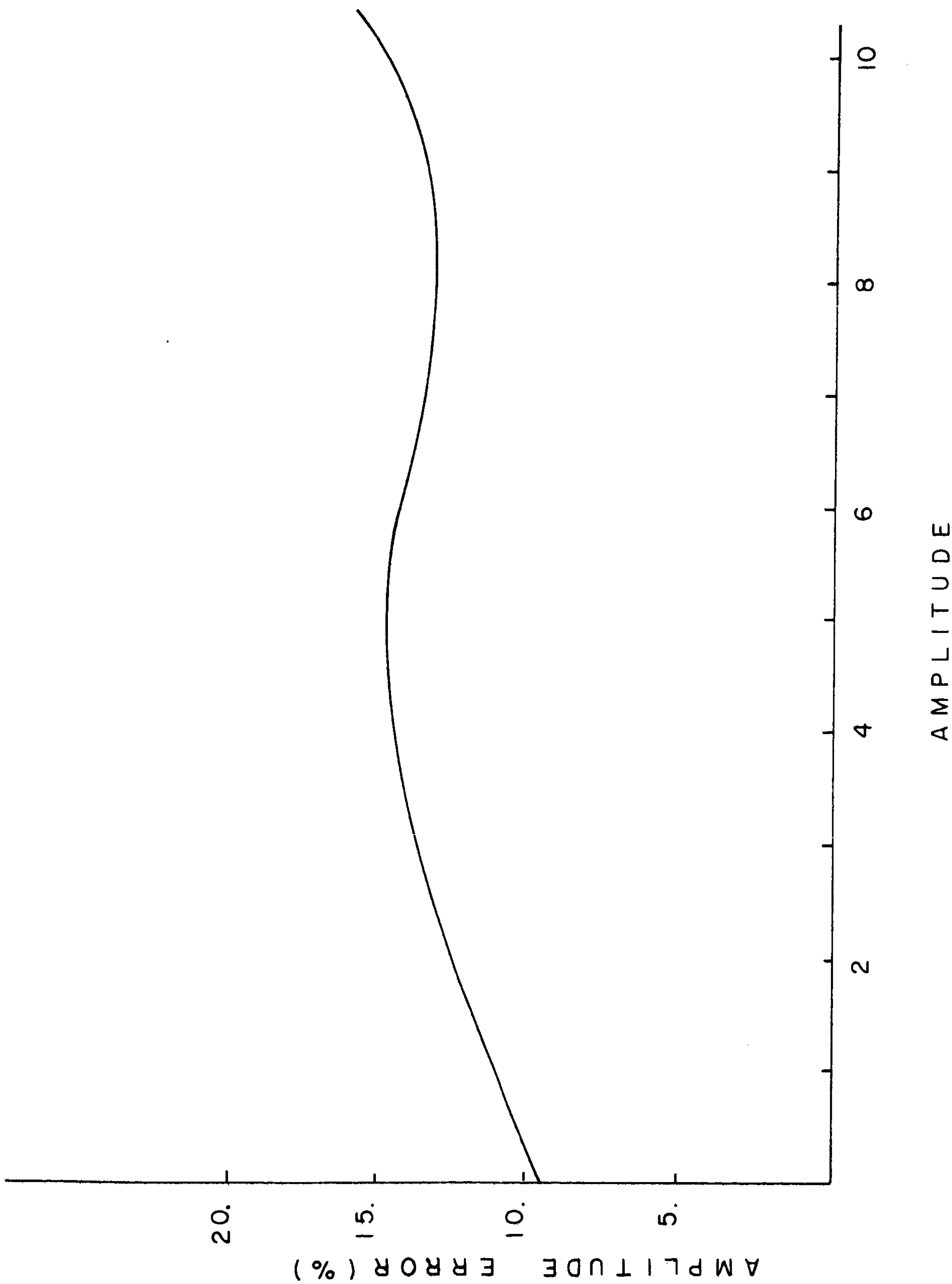

Figure 9 Errors in Amplitude Approximation

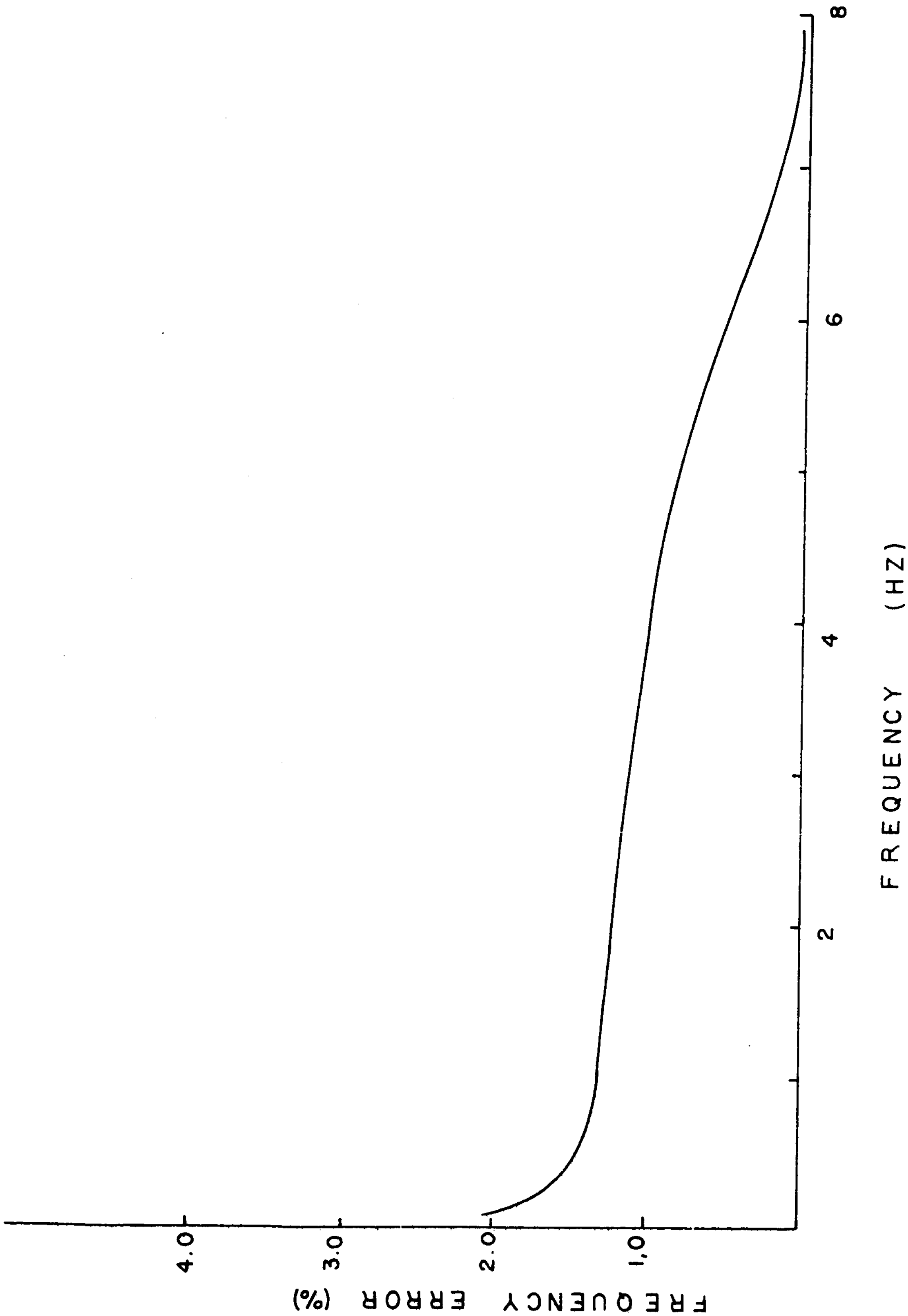

Figure 10 Errors in Frequency Isolation

greatest errors in determining ϕ occurred when ϕ was small, because of the additional phase term $2\pi \frac{N}{2} \Delta fT$ as explained in appendix C. This term gave no errors in magnitude estimation of the input, but when ϕ is small, both terms are essentially of equal strength, thus making it difficult to determine ϕ accurately.

Study of the system in uncorrelated gaussian noise yielded the results illustrated in Figures 11a and 11b. The errors shown are relative to those errors incurred when the system is operating with no input noise, i.e. the errors given would be added to those given in Figures 9 and 10.

The effect of the Hanning Window technique was to decrease all sidelobes such that mode frequencies could be displayed separately without ambiguity. Although frequency information was preserved after smoothing, the amplitudes of the maxima were attenuated; the amount of attenuation was dependent on the mode amplitude. This result may be explained by recalling that theoretically nulls should occur in the spectrum at positions corresponding to $2n\pi/(N+1)T$ units from the maximum. For the system as designed these nulls should fall at multiples of 9 storage positions from the maximum. Because of quantization error, and the fact that a finite number of frequencies were used for reference, these positions were nonzero; thus the technique tended to reduce the maximum. Large amplitudes yielded the greatest attenuation after smoothing by as much as 20%. Amplitudes less than .5 yielded less than 5% additional attenuation.

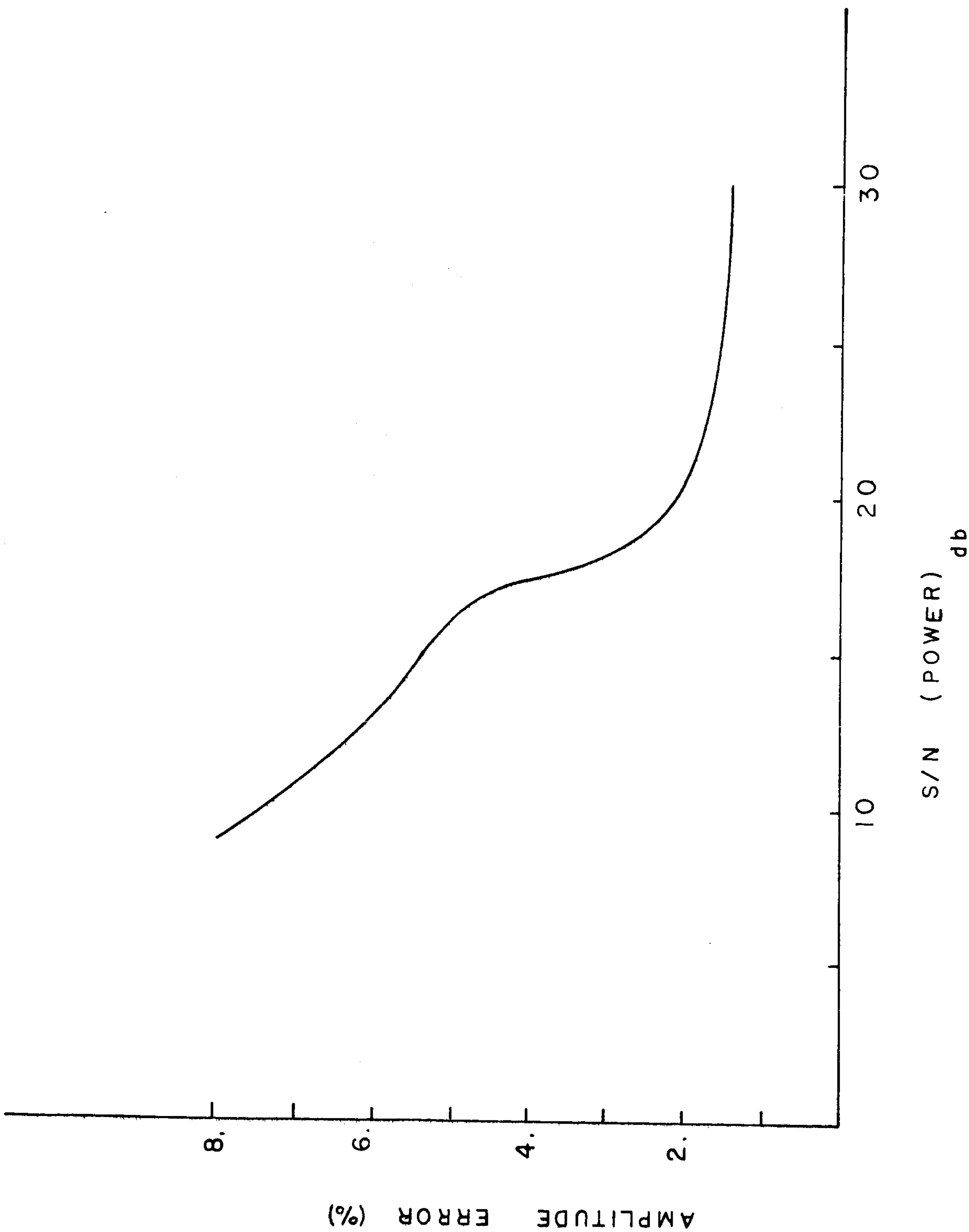

Figure 11a Errors in Amplitude with Additive Noise

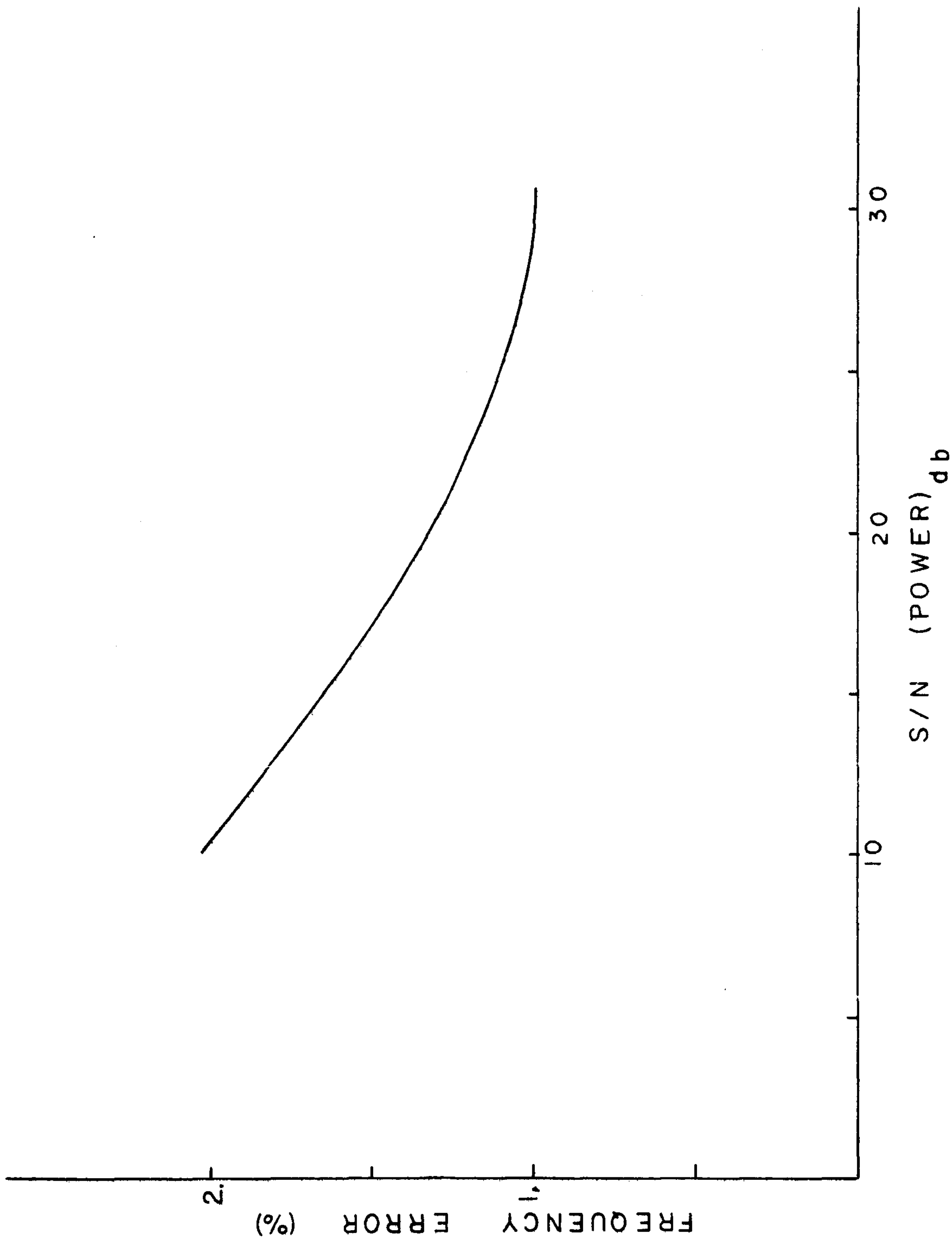

Figure 11b Errors in Frequency with Additive Noise

PART V

DISCUSSION

The system has proved capable of isolating frequencies to within the predicted limit of error. Amplitude errors are a direct result of the quantization process. To achieve greater accuracy in amplitude, it will be necessary to decrease the size of the quantization interval. If errors of greater than 5% are acceptable in frequency isolation, the system may be simplified by supplying fewer reference frequencies; however, the difference in reference frequencies (denoted Δ f in appendix C) cannot be too great or resonance will never occur.

Figures 11a and 11b have shown the behavior of the coherent memory filter in uncorrelated noise. Because the noise samples are uncorrelated, they are acted upon incoherently, in contrast to the pure input. For this reason, uncorrelated noise has little effect until its power contribution approaches that of the pure input. As a result, noise should be greater than 15 db below the pure signal.

The table look-up method of frequency generation yields far better results, as expected, than the alternate method which computed new values for each sample period. Because there are no quantization errors in the frequency generator after the initial 400 values are stored, all samples are processed more efficiently to yield a more symmetric approximation to the input spectrum.

The smoothing results show that a tradeoff is required. Since frequencies may be better displayed in a short period of time using the Hanning Window technique, the additional amplitude errors must be accepted. However, to yield better amplitude information, it is possible to store both the smoothed and unsmoothed spectrum. Thus, once mode frequencies have been isolated, the storage locations corresponding to the four mode frequencies in the unsmoothed spectrum can be examined to determine the amplitudes. The only other alternative to alleviate the problem, would be to increase the number of recirculations, thus allowing alonger process time. As shown in appendix B, the longer the process time, the more reduced are the sidelobes, and the better the spectral approximation.

PART VI

SYSTEM SPECIFICATIONS

Specifications given in this section are to be applied to the Saturn V/S-IC booster. The system will be capable of isolating bending modes one through four in frequency and amplitude. Errors in frequency display will be a maximum of 5% in the expected frequency range of .8 Hz to 8.0 Hz. The specifications are based on a normalized input, i.e. the minimum storage necessary to isolate frequencies within 5%. The maximum amplitudes of the sensors will dictate the final storage requirements.

Input Data

Mode data will be supplied for 2.8 seconds, such that 46 (N+1) sample values may be processed. If it is desired to study more than one section of the vehicle during this period, provision must be made for staggering the samples on a time multiplexed basis. Noise levels should be kept a minimum of 15 db below the signal.

Analog - Digital (A/D) Conversion

Input data will be sampled at the rate of 16 Hz, or a sampling period of .0625 seconds. Sampled data will be quantized to ten bit words (including the sign bit) at a minimum rate of 16 Hz. All samples shall be synchronized with the entire system by an external clock.

Correlation Procedure

Provision shall be made for separate correlation (two multipliers) of the quantized samples with 201 sine and cosine reference values. This will require a minimum multiply time of 80 microseconds (.0625/201) or a multiply rate of 12.5 KHz. If more than one location in the vehicle is to be processed, the multiply rate must be increased. Each multiplier will have two ten bit inputs and one ten bit output.

Frequency Generator

The frequency generator shall be capable of storing 400 ten bit reference cosines in the range 0.0 to 8.0 Hz, computed in .04 Hz frequency "steps". Selection of all cosines and sines used for correlation will follow the table look-up procedure described by equations (18) through (27) and illustrated in Figure 7. Selection of these values must be done at the minimum rate of 12.5 KHz. Values for each reference frequency will be selected every .0625 seconds (the sampling period).

If input data is multiplexed, the same reference values will be used for each location studied; as a result the multiply rate must then be increased.

Summation and Recirculation

Correlated data shall be summed with previous samples delayed by T, 2T, ... NT at the minimum rate of 12.5 KHz. Two adders are necessary for sine and cosine correlation. Each adder shall have one ten bit input, one 15 bit input, and one 15 bit output.

Each delay line [2] shall be capable of storing 201 15 bit words (spectral estimates) for .0625 seconds. After 45 recirculations

(2.8 seconds), data from each delay line will be shifted out for magnitude display. Multiplexing will require storage of 201 words per location.

Magnitude Display

After 46 input samples have been processed (45 recirculations), the 201 spectral components stored in each delay line will be further processed and displayed as a magnitude spectral estimate by the following procedure:

Divider. Each of the 201 words in the delay lines will be divided by $\frac{N+1}{2} = 23.$; the output maxima will then correspond exactly to the input amplitudes. Each divider will have a 15 bit input and a ten bit output.

Squaring. Each component (201) of the two delay lines will then be squared. The squaring device will have a ten bit input and a 20 bit output.

Summation. The nth component ($n = 1 \ldots . 201$) from each delay line will then be added. The adder will have a 20 bit input and a 21 bit output. Summation will be synchronized with each delay line.

Square Root. The square root device will have a 21 bit input and a ten bit output.

Magnitude Storage. The magnitudes of the 201 spectral components will first be smoothed by the algorithm previously discussed, i.e.

$$F(f_n) = -.5\,F(f_{n-9}) + F(f_n) - .5\,F(f_{n+9})$$

$n = 1 \ldots\ldots 201$

where $F(f_n)$ is the n^{th} spectral magnitude.

This smoothing procedure requires processing of three components by a shifting and adding procedure, i.e. multiplication by .5 is equivalent to shifting one digit to the right in the register; ten bit accuracy must still be maintained after smoothing.

The smoothed result is then stored as the n^{th} spectral magnitude estimate ($n = 1 \ldots\ldots 201$), accurate to ten bits. These magnitudes may then be scanned to yield four mode frequencies and four amplitude estimates. The n^{th} ten bit word corresponds to a spectral amplitude estimate of frequency component, $f_n = .04\ (n-1)$. If desired, the unsmoothed spectral components may then be examined to yield more accurate amplitude information.

LITERATURE CITED

1. Arden, D. N. An Alternate Form of the CMF. Raytheon Memo No. DA-66-18.

2. Blackman, R. B. Data Smoothing and Prediction. Addison-Wesley Publishing Co., Inc. 1965.

3. Blair, J. C., Lovingood, J. A., and Geisselaer, E. D. Advanced Control Systems for Launch Vehicles. Astronautics and Aeronautics, Aug. 1966.

4. Caroll, S. N. Analytical Determination of Transfer Functions for RC Commutated Networks. National Aeronautics and Space Administration. NASA TND-2948.

5. Eckberg, A. A Mathematical Analysis of the Coherent Memory Filter. Raytheon Memo AEE-1-67.

6. Eckberg, A. A Better Scheme for Generating Sampled Sine and Cosine Waves. Raytheon Memo AEE-5-67.

7. Englander, I. S. Hardware Requirements, CMF. Raytheon, Memo. ISE-1-67.

8. Englander, I. S. Further Comments on Iterative Sine Wave Generation. Raytheon, Memo. JE-9-67.

9. Evans, J. E. Digital Coherent Memory Filter. Raytheon Memo. JE-2-66.

10. Evans, J. E. Equivalence of Digital Coherent Memory Filter to a Bank of Bandpass Filters. Raytheon Memo. JE-3-66.

11. Evans, J. E. Alternate Configuration of the Digital Coherent Memory Filter Using Discrete Frequency Variations. Raytheon Memo. JE-5-66.

12. Evans, J. E. Digital Requirements for the Coherent Memory Filter. Raytheon Memo. JE-9-67.

13. Evans, J. E. A Heuristic Introduction to the Digital Coherent Memory Filter. Raytheon Memo. JE-7-67.

14. Groginsky, H. L. The Coherent Memory Filter. Raytheon Electronics Progress. Vol. 9. No. 3, pp. 7-13.

APPENDIX A

Derivation of Coherent Memory Filter Output

Consider the frequency generator to be of the form cos $w(t)t$, where $w(t) = \frac{2\pi t}{T^2}$ periodic in T, the delay line length (see fig. 4). Using complex notation:

$$m(t) = \cos\left[w(t)t\right] = \frac{1}{2}\left[e^{jw(t)t} + e^{-jw(t)t}\right]$$

$$x(t) = A\cos w_o t = \frac{A}{2}\left[e^{jw_o t} + e^{-jw_o t}\right]$$

The output $y^*(t)$ after N circulations:

$$y^*(t) = \sum_{k=0}^{N} x(t - kT)\, m(t - kT) \qquad \text{(A1)}$$

$$= \frac{A}{4}\sum_{k=0}^{N}\left[e^{jw_o(t - kT)} + e^{-jw_o(t - kT)}\right]$$

$$\times \left[e^{jw(t - kT)(t - kT)} + e^{-jw(t - kT)(t - kT)}\right] \qquad \text{(A2)}$$

The output function $y^*(t)$ will be examined from $t = 0$ to $t = T$. Since $w(t)$ is periodic in T, $w(t - T) = w(t - 2T) \ldots = w(t - NT) = w(t)$ if t varies from 0 to T.

$$w(t) = \frac{\pi}{T^2}\, t \text{ periodic in T} \qquad \text{(A3)}$$

and $\cos w(t - kT)(t - kT) = \cos \frac{\pi}{T^2} t (t - kT)$ (A4)

$$y^*(t) = \frac{A}{4} \sum_{k=0}^{N} \left[e^{jw_o(t - kT)} + e^{-jw_o(t - kT)} \right]$$

$$\times \left[e^{j\frac{\pi t}{T^2}(t - kT)} + e^{-j\frac{\pi t}{T^2}(t - kT)} \right] \qquad \text{(A5)}$$

$t = 0 \quad T$

$$y^*(t) = \frac{A}{4} \left[\sum_{k=0}^{N} e^{j(w_o + \frac{\pi t}{T^2})(t - kT)} \right. \qquad \text{A6a)}$$

$$+ e^{j(w_o - \frac{\pi t}{T^2})(t - kT)} \qquad \text{(A6b)}$$

$$+ e^{j(-w_o + \frac{\pi t}{T^2})(t - kT)} \qquad \text{(A6c)}$$

$$\left. + e^{j(w_o - \frac{\pi t}{T^2})(kT)} \right] \qquad \text{(A6d)}$$

Recalling that:

$$\sum_{k=0}^{N} e^{-jk\psi} = \frac{\sin \frac{N+1}{2}(\psi)}{\sin \frac{1}{2}(\psi)} e^{-j\frac{N}{2}(\psi)} \qquad \text{(A7)}$$

Using equation (A7) to simplify (A6):

(A6a) $$= \frac{A}{4} \; \frac{\sin \frac{N+1}{2} (w_o T + \frac{\pi t}{T})}{\sin \frac{1}{2} (w_o T + \frac{\pi t}{T})} \; e^{-j(\frac{N}{2} \; w_o T + \frac{\pi t}{T})} \; e^{j(w_o t + \frac{\pi t^2}{T^2})}$$

(A6b) $$= \frac{\sin \frac{N+1}{2} (w_o T - \frac{\pi t}{T})}{\sin \frac{1}{2} (w_o T - \frac{\pi t}{T}} \; e^{-j \; \frac{N}{2} (w_o T - \frac{\pi t}{T})} \; e^{j(w_o T - \frac{\pi t^2}{T^2})}$$

(A6c) $$= \frac{\sin \frac{N+1}{2} (-w_o T + \frac{\pi t}{T})}{\sin \frac{1}{2} (-w_o T + \frac{\pi t}{T} \;)} \; e^{-j\frac{N}{2} \; (-w_o T + \frac{\pi t}{T^2})} \; e^{j(-w_o t + \frac{\pi t^2}{T^2})}$$

(A6d) $$= \frac{\sin \frac{N+1}{2} (-w_o T - \frac{\pi t}{T}}{\sin \frac{1}{2} (-w_o T - \frac{\pi t}{T}} \; e^{-j\frac{N}{2} \; (-w_o T - \frac{\pi t}{T})} \; e^{j(-w_o t - \frac{\pi t^2}{T^2} \;)}$$

Since

$$\frac{\sin \frac{N}{2} x}{\sin \frac{x}{2}} = \frac{\sin \frac{N}{2} (-x)}{\sin \frac{1}{2} (-x)}$$

equations (A6a) and (A6c), (A6b) and A6d) may be combined to form a real time function:

$$y^{*}(t) = \frac{A}{2} \; \frac{\sin \frac{N+1}{2} (w_o T + \frac{\pi t}{T}}{\sin \frac{1}{2} (w_o T + \frac{\pi t}{T})} \; \cos(w_o T + \frac{\pi t^2}{T^2} - \frac{N}{2} \; w_o T - \frac{N\pi}{2T} \; t)$$

$$+ \frac{A}{2} \; \frac{\sin \frac{N+1}{2} (-w_o T + \frac{\pi t}{T})}{\sin \frac{1}{2} (-w_o T + \frac{\pi t}{T})} \; \cos(-w_o t + \frac{\pi t^2}{T^2} + \frac{N}{2} \; w_o T - \frac{N\pi}{2T} \; t) \qquad \text{(A8)}$$

Maxima of $y^*(t)$ occur when $w_oT = \frac{\pi t}{T}$ or $w_oT = -\frac{\pi t}{T}$. There is thus a direct relationship between the time the output maxima occur and the frequency of the input, i.e.

$$w_o = \frac{\pi t}{T^2} \tag{A9a}$$

$$w_o = \frac{\pi t}{T^2} \tag{A9b}$$

APPENDIX B

Alternate Derivation of the CMF Spectral Approximation[1]

Consider the output of the coherent memory filter of Figure 3 using $e^{jw(t)t}$ for correlation:

$$w(t) = \frac{\pi t}{T^2} \tag{B1}$$

$$y(t) = \sum_{k=0}^{N} x(t-kT)\, e^{j \frac{\pi}{T^2} (t^2 - 2kTt + k^2T^2)} \tag{B2a}$$

$$= \sum_{k=0}^{N} x(t-kT)\, e^{j \frac{\pi}{T^2} (t^2 - 2kTt)} \tag{B2b}$$

$$= e^{j \frac{\pi}{T^2} t^2} \sum_{k=0}^{N} x(t - kT)\, e^{-j \frac{2\pi kt}{T}} \tag{B2c}$$

If

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dw e^{jwt} \; X(w) \tag{B3}$$

then

$$y(t) = e^{j \frac{\pi t^2}{T^2}} \sum_{k=0}^{N} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-j \frac{2\pi kt}{T}} \, e^{jw(t - kT)} \, X(w)\, dw \tag{B4a}$$

$$= e^{j\frac{\pi t^2}{T^2}} \frac{1}{2\pi} \int_{-\infty}^{\infty} dw e^{jwt} X(w) \sum_{k=0}^{N} e^{-j(\frac{2\pi t}{T} + wT)k} \tag{B4b}$$

Since

$$\sum_{k=0}^{N} e^{-jk\psi} = \frac{\sin \frac{N+1}{2}\psi}{\sin \frac{1}{2}\psi} e^{-j\frac{N}{2}\psi} \tag{B5}$$

$$y(t) = e^{j\frac{\pi t^2}{T^2}} \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} dw e^{jwt} X(w) \frac{\sin \frac{N+1}{2}(\frac{2\pi t}{T} + wT)}{\sin \frac{1}{2}(\frac{2\pi t}{T} + wT)}$$

$$x e^{-j\frac{N}{2}(\frac{2\pi t}{T} + wT)} \tag{B6}$$

For N large, the function

$$\frac{\sin \frac{N+1}{2}(\frac{2\pi t}{T} + wT)}{\sin \frac{1}{2}(\frac{2\pi t}{T} + wT)}$$

can be considered as a sequence of impulses with argument w and area 1 centered at values of w given by

$$w_n T + \frac{2\pi t}{T} = 2n\pi \tag{B7a}$$

$$w_n = \frac{-2\pi t}{T^2} + \frac{2n\pi}{T} \tag{B7b}$$

Since $|w| \leq \frac{\pi}{T}$ for unambiguous representation, $X(w) = 0$ for $|w| \geq \frac{\pi}{T}$. Therefore, only one impulse centered at $w = \frac{\pi t}{T^2}$ occurs.

The output thus becomes:

$$y(t) \cong X\left(\frac{-2\pi t}{T^2}\right) \tag{B8a}$$

an approximation to the negative input spectrum.

If

$e^{jw(t)t}$

were replaced by

$e^{-jw(t)t}$

$$y(t) \cong X\left(\frac{2\pi t}{T^2}\right) \tag{B8b}$$

an approximation to the positive input spectrum.

Using both cosine and sine reference values, the magnitude of any spectrum between $\pm \frac{1}{2T}$ may be displayed with large enough N.

APPENDIX C

Derivation of the Digital Coherent Memory Filter Output.

Consider the resultant output for an input of the form

$$x(t) = \cos(2\pi f_1 t + \phi) \qquad \text{(C1)}$$

Using cosine correlation the output becomes the following:

$$F_1(f_n) = \sum_{k=0}^{N} \cos(2\pi f_1 kT + \phi)\cos(2\pi f_n kT) \qquad \text{(C2)}$$

$$n = 1 \;\ldots\ldots\; M+1$$

$$= \frac{1}{4} \sum_{k=0}^{N} \left[e^{j(2\pi f_1 kT + \phi)} + e^{-j(2\pi f_1 kT + \phi)} \right]$$

$$x \left[e^{j(2\pi f_n kT)} + e^{-j(2\pi f_n kT)} \right] \qquad \text{(C3)}$$

$$= \frac{1}{4} \sum_{k=0}^{N} e^{j\left[2\pi(f_1 + f_n)\, kT + \phi\right]} + e^{j\left[2\pi(-f_1 + f_n)\, kT - \phi\right]}$$

$$+ e^{j\left[2\pi(f_1 - f_n)kT + \phi\right]} + e^{-j\left[2\pi(f_1 + f_n)kT + \phi\right]} \qquad \text{(C4)}$$

Proceeding as in appendix A, the output becomes

$$F_1(f_n) = \frac{1}{2} \frac{\sin \frac{N+1}{2}\,(2\pi(f_1 + f_n)\, T}{\sin \frac{1}{2}\quad 2\pi(f_1 + f_n)\, T} \cos\left[\frac{N}{2}\, 2\pi(f_1 + f_n)T + \phi\right] \qquad \text{(C5a)}$$

$$+ \frac{1}{2} \frac{\sin \frac{N+1}{2} \quad 2\pi(f_1 - f_n) T}{\sin \frac{1}{2} \quad 2\pi(f_1 - f_n) T} \cos \left[\frac{N}{2} 2\pi(f_1 - f_n) T + \phi \right] \qquad \text{(C5b)}$$

Equation (C5a) has essentially zero contribution except where $f_1 + f_n = \frac{1}{T}$. This is possible only if f_1 and f_n are $\geq \frac{1}{2T}$. Frequencies are being supplied from 0 to $\frac{1}{2T}$; therefore equation (C5a) may be neglected.

Equation (C5b) has a maximum when $f_1 = f_n$, i.e. resonance. Because only a finite number of reference frequencies are supplied, f_1 may never exactly equal f_n. At worst:

$$f_1 = f_n + \frac{\Delta f}{2}$$

where Δf is the difference between reference frequencies.

At worst then:

$$F_1(f_n) = \frac{1}{2} \frac{\sin \frac{N+1}{2} \left(\frac{\Delta f}{2} T \right)}{\sin \frac{1}{2} \left(\frac{\Delta f}{2} T \right)} \cos \left[\frac{N}{2} 2\pi \frac{\Delta f}{2} T + \phi \right] \qquad \text{(C6)}$$

For $\frac{\Delta f}{2}$ small:

$$F_1(f_n) = \frac{1}{2} \cos \phi \qquad \text{(C7)}$$

If sine correlation were used the output would be

$$F_2(f_n) = - \frac{1}{2} \sin \phi \qquad \text{(C8)}$$

Therefore, magnitudes may displayed as in the analog case.

APPENDIX D

Noise Response Characteristics

If uncorrelated noise samples of variance σ_o and mean 0 are processed by the coherent memory filter, the resulting words stored in the delay line correspond to an approximation of each frequency component between $\pm\frac{1}{2T}$. The variance of each component y_m may be characterized by the following equation:

$$\mathrm{Var}(y_m) = \sum_{k=0}^{N} \left[n_{kT} \cos 2\pi f_m kT\right]^2 \tag{D1}$$

where y_m is the m^{th} frequency component, n_{kT} is the k^{th} noise sample.

$$\mathrm{Var}(y_m) = \sum_{k=0}^{N} n_{kT}^2 \cos^2 2\pi f_m kT \tag{D2}$$

since the noise samples are uncorrelated.

$$\cos^2 2\pi f_m kT \leq 1$$

$$\therefore \quad \mathrm{Var}(y_m) \leq \sum_{k=0}^{N} n_{kT}^2 \tag{D3a}$$

$$\leq \sum_{k=0}^{N} \sigma_o^2 \tag{D3b}$$

$$\mathrm{Var}\,(y_m) \leq (N+1)\,\sigma_o^2 \tag{D4}$$

standard deviation of y_m, σ_y becomes

$$\sigma_y \leq \sigma_o \sqrt{N+1}$$

The rms value of an input $x(t) = A\cos w_o t$, uncorrupted by noise is

$$x_{rms} = \frac{A}{\sqrt{2}}$$

The input signal to noise power ratio becomes:

$$(S/N)_i = \frac{A^2}{2\sigma_o^2} \tag{D5}$$

The rms value of the output uncorrupted by noise is

$$y_{rms} \leq \frac{A(N+1)}{2}$$

$$(S/N)_o \leq \frac{A^2 \; N+1}{2} \quad \sigma_o^2(N+1) \tag{D6}$$

Improvement in signal to noise power ratio becomes:

$$\frac{(S/N)_o}{(S/N)_i} = \frac{\frac{A^2(N+1)^2}{2} \quad \sigma_o^2(N+1)}{A^2/2\,\sigma_o^2} = \sqrt{(N+1)} \tag{D7}$$

The peak signal to noise ratio improvement becomes:

$$\frac{(S/N)_o}{(S/N_i)_i} = \sqrt{N+1} \tag{D8}$$

Thus, the major effect of noise is to increase the sidelobes of the output function.

APPENDIX E

The graph on the following page describes the variation of mode frequencies with time. They may vary from that indicated by $\pm$ 20%. The minimum separation of modes is .75 Hz. The maximum rate of change of frequency with time is .2 Hz/sec. occurring in the fourth bending mode. Data was taken from Boeing Report No. D5-1538-1.

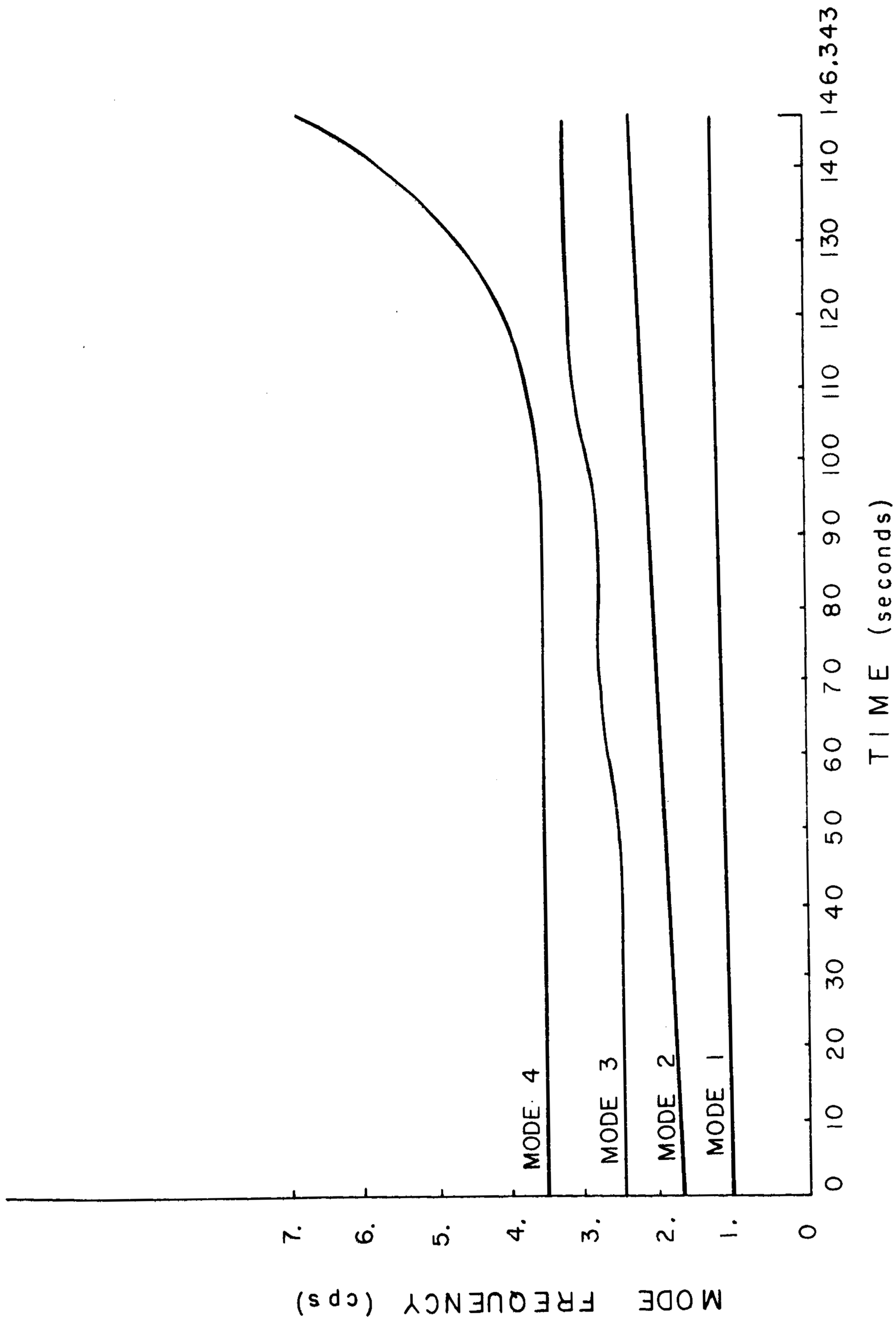

Figure E1 Mode Frequency Variation During Flight

APPENDIX F

Derivation of Number of Recirculations N

Consider the behavior of the coherent memory filter if the maximum of the spectrum of one mode occurred at the first null of another mode:

$$\frac{\sin \frac{N+1}{2} (w_o T - x)}{\sin \frac{1}{2} (w_o T - x)} = 0 \qquad x = w_o T - \frac{2\pi}{N+1} \qquad \text{(F1)}$$

$$\frac{\sin \frac{N+1}{2} (w_1 T - x)}{\sin \frac{1}{2} (w_1 T - x)} = N+1 \qquad x = w_o T - \frac{2\pi}{N+1} = w_1 T \qquad \text{(F2)}$$

$$\therefore \quad w_1 T = w_o T - \frac{2\pi}{N+1}$$

$$(w_o - w_1)\, T = \frac{2\pi}{N+1}$$

$$f_o - f_1 = \frac{1}{(N+1)T} \qquad \text{(F3)}$$

A minimum resolvable frequency difference is therefore $\frac{1}{(N+1)T}$.

For N=45, $f_o - f_1$ must be $\leq .36$ Hz. From data included in Boeing Report No. D5-15381-1, mode frequencies differ by .75 Hz or greater; therefore, all modes may be adequately isolated for an N of 45.

```
C     DESCRIPTION OF SYMBOLS
C     SIMULATION OF THE LOW FREQUENCY DIGITAL COHERENT MEMORY FILTER
C     A(I)=AMPLITUDE OF I TH BENDING MODE
C     F(I)=FREQUENCY OF I TH BENDING MODE
C     PHI(I)=PHASE SHIFT OF I TH MODE,  READ IN AS A MULTIPLE OF PI
C     AMODE=TOTAL MODE AMPLITUDE
C     T=DELAY LINE LENGTH=.0625 SECONDS
C     REFNUM= MAXIMUM NUMBER OF REFERENCE FREQUENCIES=400.
C     N=NUMBER OF RECIRCULATIONS
C     NREF=NUMBER OF REFERENCE FREQUENCIES STORED FOR SELECTION=400
C     NFREQ=NUMBER OF REFERENCE FREQUENCIES USED FOR CORRELATION=201
C     NQUANT=NUMBER OF QUANTIZATION LEVELS FOR CORRELATION VALUES
C      Q=NUMERICAL VALUE OF ONE QUANTIZATION INTERVAL=1/2 TO THE
C     NQUANT POWER
C     COSREF=STORED REFERENCE VALUES FROM WHICH CORRELATION VALUES ARE
C     SELECTED
C     COSMIX=VALUES USED FOR COSINE CORRELATION
C     SINMIX=VALUES USED FOR SINE CORRELATION
C     INDEX=REFERENCE VALUE LOCATION IN THE STORED MATRIX
C     LAG=NUMBER OF REFERENCE POSITIONS CORRESPONDING TO PI/2
C     LEAD=NUMBER OF REFERENCE POSITIONS CORRESPONDING TO 3PI/2
C     OUTPUT(KK,1)=KK TH SPECTRAL COMPONENT
C     OUTPUT(KK,2)=AMPLITUDE OF KK TH SPECTRAL COMPONENT
C     YCOS=OUTPUT OF DELAY LINE USED FOR COSINE CORRELATION
C     YSIN=OUTPUT OF DELAY LINE USED FOR SINE CORRELATION
      DIMENSION YCOS(201),YSIN(201),COSMIX(201),SINMIX(201),INDEX(201)
      DIMENSION COSREF(400),OUTPUT(201,2)
      DIMENSION A(4),F(4),PHI(4),RAD(4)
      READ(1,1) T,REFNUM,N,NREF,NFREQ,LAG,LEAD,NQUANT
    1 FORMAT(2F10.4,6I10)
      AN=N
      QUANT=NQUANT
      Q=1./QUANT
      TPI=2.*3.1415927
      READ(1,30) A,F,PHI
   30 FORMAT(8F10.4)
      DO 31 I=1,4
      PHI(I)=PHI(I)*3.1415927
   31 RAD(I)=TPI*F(I)
C     INTIALIZE CORRELATION VALUES AND DELAY LINES
      DO 10 I=1,NFREQ
      COSMIX(I)=1.
      SINMIX(I)=0.
      YSIN(I)=0.
   10 YCOS(I)=0.
C     COMPUTE REFERENCE COSINES
      DO 26 M=1,NREF
      AM=M-1
      COSREF(M)=COS(TPI*AM/REFNUM)
   26 CONTINUE
C     BEGIN CORRELATION
      TIME=0.
      L=N&1
```

```
      DO 102 I=1,L
C     COMPUTE SAMPLED INPUT
      AMODE=0.
      DO 100 MM=1,4
  100 AMODE=AMODE&A(MM)*COS(RAD(MM)*TIME&PHI(MM))
C     QUANTIZE VALUES
      CALL ROUND(AMODE,Q)
C     CORRELATION VALUES FOR FIRST TWO SAMPLES ARE SELECTED DIRECTLY
C     FROM COLUMNS 1 AND2 FROM THE MATRIX SHOWN IN FIGURE 5
      IF(I-2) 200,202,201
  202 DO 205 J=1,NFREQ
      JJ=J-LAG
      JJJ=J&LEAD
      INDEX(J)=J
      COSMIX(J)=COSREF(J)
      IF(J-LAG-1)300,301,301
  301 SINMIX(J)=COSREF(JJ)
      GO TO 205
  300 SINMIX(J)=COSREF(JJJ)
  205 CONTINUE
      GO TO 200
C     BEGIN TABLE LOOKUP METHOD OF SELECTION
  201 CALL GENRAT(COSMIX,SINMIX,COSREF,NFREQ,NREF,INDEX,LAG,LEAD)
C     COMPUTE CORRELATION OF INPUT WITH SELECTED REFERENCE VALUES
  200 DO 101 K=1,NFREQ
      SININ=AMODE*SINMIX(K)
      COSIN=AMODE*COSMIX(K)
      CALL ROUND(SININ,Q)
      CALL ROUND(COSIN,Q)
C     COMPUTE INPUT TO EACH DELAY LINE
      YSIN(K)=YSIN(K)&SININ
      YCOS(K)=YCOS(K)&COSIN
      CALL ROUND(YSIN(K),Q)
  101 CALL ROUND(YCOS(K),Q)
  102 TIME=TIME&T
C     AFTER N RECIRCULATIONS PROCESS FOR MAGNITUDE DISPLAY
      DO 50 KK=1,NFREQ
      AKK=KK
      OUTPUT(KK,1)=(AKK-1.)*.04
      XOUT=2.*YSIN(KK)/(AN&1.)
      CALL ROUND(XOUT,Q)
      YOUT=2.*YCOS(KK)/(AN&1.)
      CALL ROUND(YOUT,Q)
      XSQR=XOUT*XOUT
      YSQR=YOUT*YOUT
      CALL ROUND(YSQR,Q)
      CALL ROUND(XSQR,Q)
      OUT=XSQR&YSQR
      CALL ROUND(OUT,Q)
      OUTPUT(KK,2)=SQRT(OUT)
      CALL ROUND(OUTPUT(KK,2),Q)
   50 CONTINUE
C     SMOOTH OUTPUT USING HANNING WINDOW
      CALL SMOOTH(NFREQ,OUTPUT)
      CALL EXIT
```

```
      SUBROUTINE ROUND(FUNCT,Q)
C     THIS SUBROUTINF MAINTAINS ALL VALUES IN THE SYSTEM TO DISCRETE LEVELS
C     DESCRIPTION OF SYMBOLS
C      Q=SIZE OF ONE QUANTIZATION INTERVAL
C     INTR=NUMBER OF QUANTIZATION INTERVALS CHARACTERIZING FUNCTION
      AINTR=FUNCT/Q
      IF(FUNCT)1,2,3
    1 AINTR=AINTR-.5
      GO TO 4
    3 AINTR=AINTR&.5
    4 INTR=AINTR
      AINTR=INTR
      FUNCT=AINTR*Q
    2 RETURN
      END
      SUBROUTINE SMOOTH(NFREQ,OUTPUT)
C     THIS SUBROUTINE SMOOTHS THE OUTPUT MAGNITUDE BY THE HANNING WINDOW
C     TECHNIQUE
C     OUTPUT(K,1)= K TH SPECTRAL COMPONENT
C     OUTPUT(K,2)=UNSMOOTHED SPECTRAL AMPLITUDE
C     YSMTH(K)=SMOOTHED SPECTRAL AMPLITUDE
      DIMENSION OUTPUT(201,2),YSMTH(201)
      L=NFREQ-9
      DO 10 K=10,L
      KK=K-9
      KKK=K&9
      YSMTH(K)=-.5*OUTPUT(KK,2)&OUTPUT(K,2)-.5*OUTPUT(KKK,2)
      IF(YSMTH(K))100,100,10
  100 YSMTH(K)=0.
   10 WRITE(3,22) OUTPUT(K,1),OUTPUT(K,2),YSMTH(K)
   22 FORMAT(1H 3F12.6)
      RETURN
      END

      SUBROUTINE GENRAT(Z,Y,C,NFREQ,NREF,INDEX,LAG,LEAD)
C     THIS SUBROUTINE GENERATES ALL CORRELATION COSINES AND SINES BY
C     A TABLE-LOOKUP METHOD
C     DESCRIPTION OF SYMBOLS
C     C=STORED REFERENCE VALUES
C     Y=VALUE SELECTED FOR SINE CORRELATION
C     Z=VALUE SELECTED FOR COSINE CORRELATION
      DIMENSION Z(201),Y(201),C(400),INDEX(201)
      DO 10 M=1,NFREQ
C     SELECT COSINE REFERENCE
      INDEX(M)=INDEX(M)&M-1
      IF(INDEX(M)-NREF)12,12,11
   11 INDEX(M)=INDEX(M)-NREF
   12 J=INDEX(M)
      Z(M)=C(J)
C     SELECT SINE REFERENCE
      IF(INDEX(M)-LAG-1)13,14,14
   13 J=INDEX(M)&LEAD
      GO TO 15
   14 J=INDEX(M)-LAG
   15 Y(M)=C(J)
   10 CONTINUE
      RETURN
      END
```